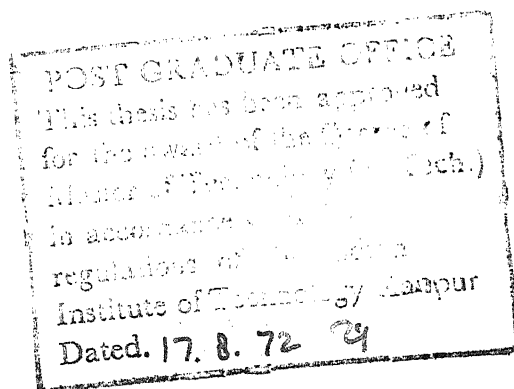


# **SOME EXPERIMENTAL STUDIES ON PULL-OUT RESISTANCE OF FOUNDATIONS**

**A Thesis Submitted  
In Partial Fulfilment of the Requirements  
for the Degree of  
MASTER OF TECHNOLOGY**

**BY  
A. SREEPADA RAO**



to the

**DEPARTMENT OF CIVIL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY KANPUR  
JUNE, 1972**

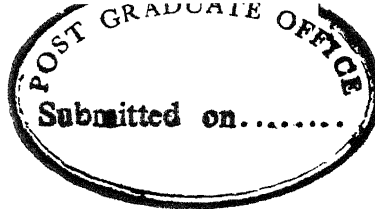
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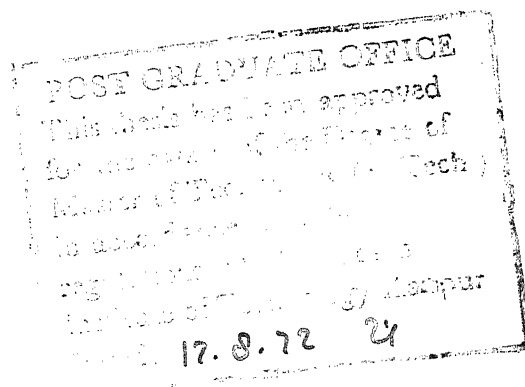
# C E R T I F I C A T E

This is to certify that the thesis entitled  
'SOME EXPERIMENTAL STUDIES ON PULL-OUT RESISTANCE OF  
FOUNDATIONS' by A. SREEPADA RAO is a record of work  
carried out under my supervision, and has not been  
submitted elsewhere for a degree.

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A. SREEPADA RAO

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## LIST OF SYMBOLS

$A$	=	Area of a footing
$A_r$	=	Area ratio of a footing
$B$	=	Diameter of a circular footing; Side of a square footing; Short side of a rectangular footing.
$B_1$	=	Radius of a circular footing; Equivalent radius of footing of shape other than a circle.
$B_{Eq}$	=	Equivalent $B$ for footing of shape other than a circle.
$D$	=	Depth of embedment
$F_1$	=	Frictional force on sliding surface (Matsuo's method).
$F_1, F_2, F_3$	=	Functions, as for e.g. $F_1(\phi, \lambda)$ .
$K$	=	Coefficient of earth pressure
$L$	=	Length of a rectangular footing
$P$	=	Pull-out force on a footing; Pull-out resistance of foundations.
$P_1, P_2, P_b$	=	Earth pressures
$P_r$	=	Perimeter ratio of a footing
$R$	=	Radius of sliding surface
$R_f$	=	Radius of the line of rupture at the ground surface.
$T'$	=	Shearing force on rupture surface
$W$	=	Weight of soil in failure zone

$c$	=	Cohesion of soil
$q$	=	Average pull-out pressure per unit area of footing = $\frac{P}{A}$ .
$q_o$	=	Over burden pressure at the footing level = $\gamma D$ .
$\alpha, \rho'$	=	Angle of rupture surface with the vertical (Earth cone method, Mors' method).
$\delta$	=	Displacement of the footing
$\gamma$	=	Density of soil
$\gamma_z$	=	$\gamma$ at a depth $z$ from surface
$\lambda$	=	Coefficient $\frac{D}{B}$ in Balla's method and $\frac{D}{B_1}$ in Matsuo's method.
$\phi$	=	Angle of internal friction
$\theta_o$	=	Central angle of the logarithmic spiral (Matsuo's method).

## SYNOPSIS

Experiments are conducted to evaluate the resistances of footings and foundation soil to pull-out forces. The values are compared with those obtained by using the charts and expressions of Balla and Matsuo. It is found that the latter values are inaccurate for ordinary fine grain sand and are gross underestimates of the pull-out strengths of dense fine grain sand.

The effect of shape of footing is discussed vis-a-vis the common shapes in use. Densification of sand is found to increase the pull-out strengths considerably.

Results of 13 tests in ordinary sand, and 20 in dense sand, with different depths of embedments and various shapes of footing indicate a combined or common curve on a non-dimensional plot representing the behaviour of all footings for a given density of the cohesionless soil. More experiments are necessary before this conclusion can be generalized.

## CHAPTER 1

### INTRODUCTION

#### 1.1. GENERAL:

In a variety of cases, parts of foundations are subjected to pure tensile forces, and have to be designed accordingly. Under these conditions, it is the capacity of the soil to hold the foundation together and to resist the pull out forces, that comes into play; more often than not, the up-lift capacity of the soil when the parts of the foundation are subject to tension, is the governing factor of design, than the bearing capacity of the soil when these same parts are acted on by compressive forces.

Examples of situations where the problem arises are as follows:

- 1) Foundations of massive towers, as for say, transmission lines, which in the worst condition, are to be designed for horizontal pull from the wires on one side only;
- 2) High capacity water tanks of great heights, are subject to wind loads, but because of their massive structure have necessarily to be provided in many cases with individual column footings;
- 3) Under-reamed piles, and even grillage foundations which are used as anchorages;

- 4) In many marine operations, in both shallow and deep waters, anchors are used to apply upward direct forces to the ocean bottom; the problem is also encountered in the stability of surface or submerged platforms anchored to ocean bottom;
- 5) This is again a factor in the design and construction of deep sea habitats, and salvage operations on sunken ships. In this case, the problem is to find the magnitude of the force required to cause the complete withdrawal of an object of known shape, dimensions and weight, brought by some operation to rest at some depth below the ocean bottom and embedded either partially or fully.

In general, any foundation subject to tension, either as a direct force, or as the result of an overturning moment on the structure, has to depend on the capacity of the soil block surrounding it, to resist the pull out forces.

## 1.2 NEED FOR EXPERIMENTAL INVESTIGATION:

The subject of pull-out forces on foundations has not received much attention from the researchers, and earlier designs were mainly by thumb-rules, or empirical formulae. The theories proposing these empirical formulae sometimes even ignored important properties of soils. It is only in the last decade or so, that theoretical analyses (1,7,8) based on

rational principles of soil mechanics, and established procedures of theory of elasticity have been attempted. Like many other areas of soil mechanics, analyses on this topic also assume ideal homogeneous soils, with constant density and angle of friction  $\phi$  for all depths. They assume rupture surfaces and treat the essentially three dimensional problem as a two dimensional one. Further, whereas the very common foundations in practice are squares and rectangles in shape, all the analyses so far are based on circular shaped footings, and the results of these form the basis for empirical modifications for other shapes of foundations. These make it all the more essential that the values so arrived at by theories be verified experimentally both in the laboratory and in the field.

The problem assumes importance and urgency at least in one major field, that of transmission tower foundations, in the context of expanding power projects in India.

### 1.3 SCOPE OF THE INVESTIGATION:

Experiments on pull-out forces on foundations can be many-fold, and the parameters governing the performance are numerous. Soil types and properties (cohesive and cohesionless soils, anisotropic soils, non-homogeneous soils including layered deposits), variations in grain-size and densities,

the sizes, shapes and materials of models, depths of embedment, and nature or modes and direction of application of load are some of the factors which require to be taken into account in an extensive investigation project. However, the present series of experiments are limited to the following studies:-

1. To conduct pull-out tests on models of different sizes, and shapes, at different depths of embedments. Tests have been conducted on circular, square and rectangular models, and two models with strip-like projections on the periphery of a circle and a square;
2. To conduct a few of the above tests with reference to pull-out loads vs. the corresponding displacements of the foundations;
3. To qualitatively observe the rupture surface shape;
4. To study the effect of sand densification on the pull-out force on the same model, and;
5. To compare the results of the above experiments with the values given by Balla's and Matsuo's formulae and charts.



## CHAPTER 2

### LITERATURE REVIEW

#### 2.1 GENERAL REVIEW:

Early methods of estimating the pull-out forces of foundations were rather empirical and arbitrary. Some of the procedures are given below with brief comments:

##### (a) THE EARTH-CONE METHOD (4,15):

The pull-out force  $P$  is equated to the sum of the self-weight of the footing and the weight of the soil mass contained in a truncated cone between the base of the footing and the ground surface. (Figure 2.1). The angle  $\alpha$  is empirically chosen depending on the soil type.

In this method, the shear failure and cohesion of the earth body are neglected. The calculated resistance using this method, increases greatly with the depth of embedment  $D$ , contrary to actual practice.

##### (b) THE EARTH-PRESSURE METHOD (6):

The pull-out resistance  $P$  is given by

$P =$  Weight of footing

+ weight of the soil vertically above the  
footing slab

+ the frictional force on the vertical face at  
the condition of earth pressure at rest,

as shown in figure 2.2.

Here again the shear failure of the soil and the cohesion do not come into the picture.

(c) THE SHEARING METHOD:

This method takes into account the shear resistance along the failure surface, and to this extent is better than the earlier two methods. It is based on a vertical shearing surface, same as in figure 2.2. The pull-out force is the sum of the weights of the footing and the soil within the failure zone, and a shear  $T'$  on the failure surface. But unlike the previous method,  $T'$  in this is based on  $c$  and  $\phi$ , the cohesion and the friction angle of the soil respectively. Expressions for  $T'$  were given for a square slab, but have been later modified by Schichiri (11) and by the Institute of Electrical Engineering, Japan (4).

A vital draw-back of the method is that the vertical failure surface does not occur in practice.

(d) MORS' METHOD (10):

Mors takes into account both a sliding surface (figure 2.3) and the shear behaviour of soils, and finally suggests an empirical formula for practical design purposes. The angle  $\alpha'$  varies with the type and condition of the soil. But this can basically be termed the earth cone method only, in a general sense.

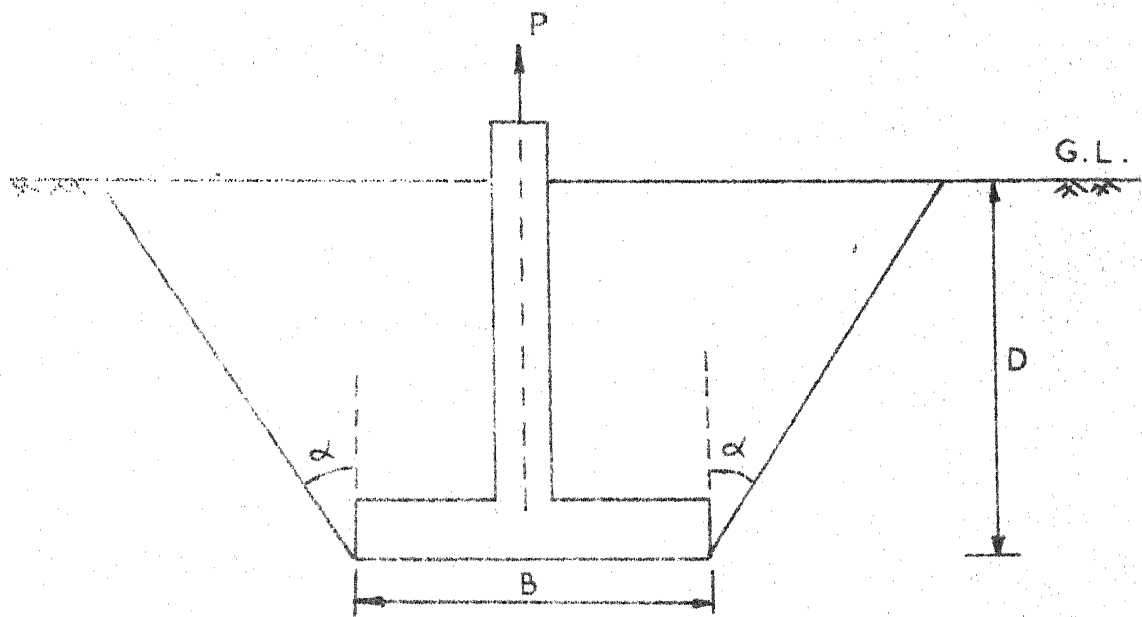


FIG. 2.1\_ THE EARTH CONE METHOD

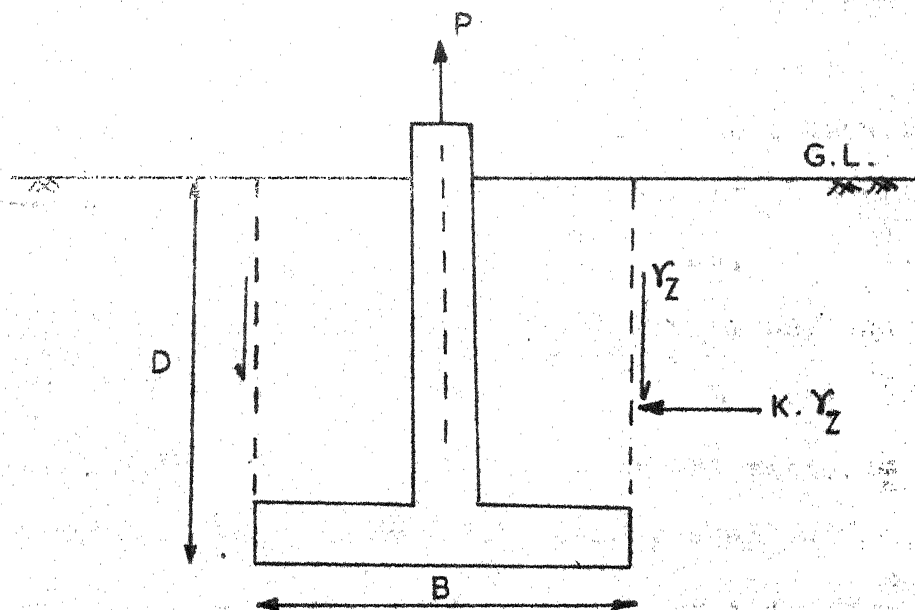


FIG. 2.2\_ THE EARTH PRESSURE METHOD. (FOR THE SHEARING METHOD, THE SHEAR IS  $T'$  BASED ON  $C$  AND  $\phi$  VALUES OF SOIL)

## 2.2 TURNER'S EXPERIMENTS (17):

The study was directed mainly to the evaluation of uplift resistance of, and to the development of design criteria for, three types of footings - under-reamed, straight shaft and grillage foundations - that were in use as tower foundations in transmission lines of the Houston Lighting and Power Co., U.S.A. The study fails, however to arrive either at a generalized theory of failure or even at a failure surface, and the design criteria therein are mainly empirical. However, the important contribution of Turner is that by laboratory and small scale prototype field tests he rejects the then prevalent theories of -

- (i) shear along a cylindrical surface,
- (ii) tension and shear along a conical surface,
- (iii) shear along a parabolic surface, and
- (iv) shear along a vertical surface near base of footing plus tension along a conical surface above the cylindrical shear zone.

Turner makes some interesting observations, and some of his conclusions are -

- a) Footings with a depth (D) to diameter (B) ratio,  $\frac{D}{B} \geq 1.5$  are deep footings, and  $\frac{D}{B} < 1.5$  means a shallow foundation,
- b) The pull-out force P for footings with  $\frac{D}{B} > 1.5$  is not significantly influenced by D, while for  $\frac{D}{B} \leq 1.5$ , P is a

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function of  $(\frac{D}{B})^2$ . The limiting value of  $\frac{D}{B} = 1.5$  defined the failure loads and soil deformations at failure.

### 2.3 BALLA'S METHOD:

Balla (1) was probably the first who attempted a rational analysis for calculating the anchor pull-out force  $P$ , taking into account the soil properties, both shear and cohesion. The pull-out force is obtained as the sum of the dead weights of the footing and the breaking out earth, and the vertical component of the resultant shear stress acting on the sliding surface, which is taken as an arc of a circle (Fig. 2.4). The conclusions from his analysis however, are incorrect at least in one aspect. While it is true that  $P$  depends on soil characteristics (density  $\gamma$ , friction angle  $\phi$  and cohesion  $c$ ), and on what he terms as formal coefficient  $\lambda = \frac{D}{B}$ , Balla's formula gives  $P$  as proportional to  $D^3$  ( $D$  = Depth of embedment), i.e.  $P$  increases greatly with  $D$ , which is not true in practice and as evidenced in Turner's tests also. Even the limited number of Balla's laboratory tests reported, do indicate that at low values of  $D$  the test values of  $P$  are higher, and at higher values of  $D$ ,  $P$  is lower than his own theoretical values. Balla ignores these discrepancies probably as within limits of experimental errors, and claims reasonable agreement between his theory and the test values.

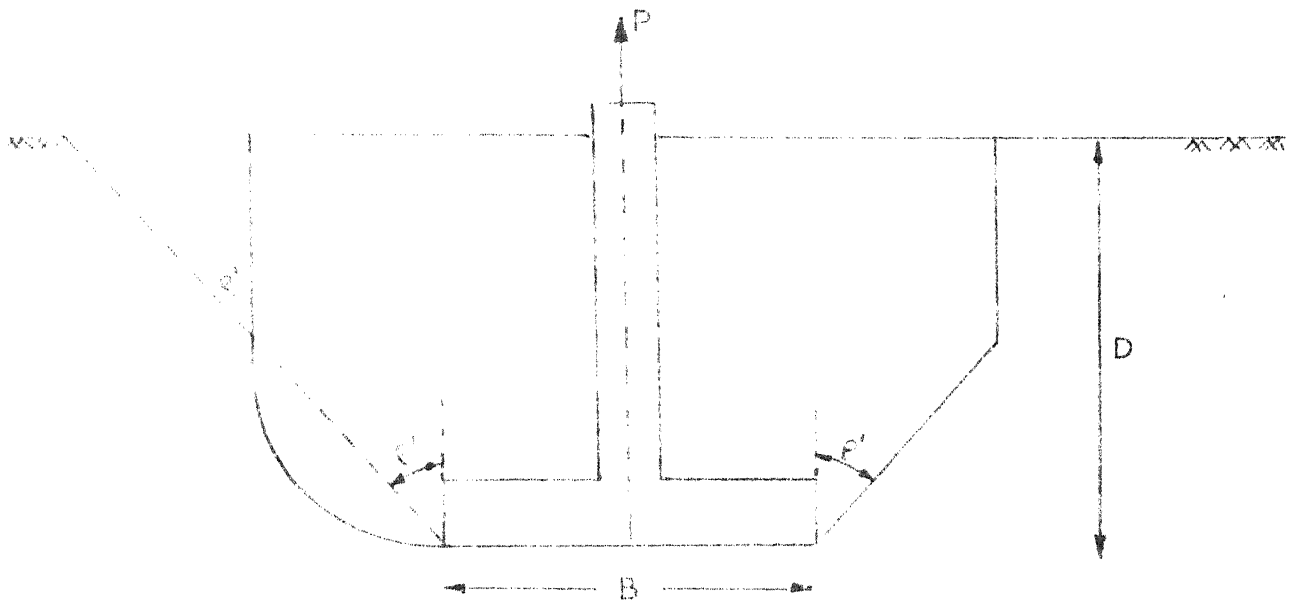


FIG. 2.3. MORS' METHOD

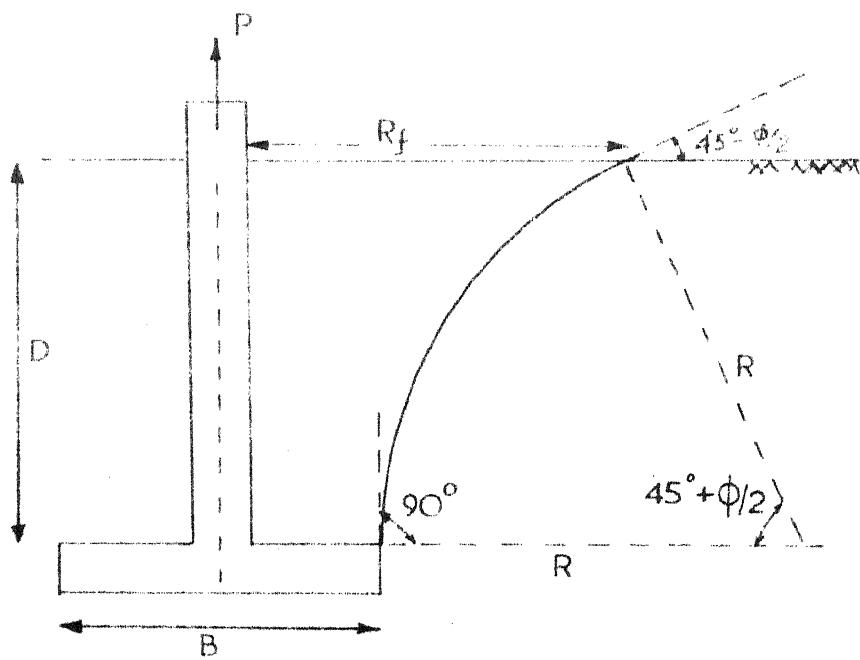


FIG. 2.4 - BALLA'S METHOD

## 2.4 MEYERHOFF AND ADAM'S METHOD (9):

A theory is derived for a long continuous footing and modifications are suggested for other shapes. According to them, no rigorous analysis is available for the determination of the sliding surface, and they resort back to the earth pressure method (2.1.(b)) already mentioned, with just this difference that it is the coefficient of passive pressure taken instead of that at rest. The agreement between theoretical and observed values is not so good; but a significant contribution of Meyerhoff and Adams is a maximum value of  $D/B$  determined for a given angle  $\phi$  beyond which further depth of embedment does not contribute to an increase in strength. (Table 2.1). The limiting values of this ratio are based on considerable experimental data.

---

TABLE 2.1: LIMITING VALUES OF  $D/B$  BEYOND WHICH NO INCREASE IN UPLIFT RESISTANCE IS OBSERVED.

---

$\phi$ in degrees	20	25	30	35	40	45	48
Limiting $D/B$ ratio	2.5	3.0	4.0	5.0	7.0	9.0	11.0

---

## 2.5 MATSUO'S METHOD (7,8):

Matsuo assumes the failure surface as a logarithmic

spiral and in continuation a tangential straight line corresponding to Rankine passive earth pressure state of stress (Figure 2.5). The trial and error procedure for the determination of the slip surface is similar to Terzaghi's passive earth pressure concept (16). The pull-out force is then given as the sum of the weight of the footing and of the breaking out soil mass, together with the vertical component of the resultant shear resistance acting on the sliding surface.

Matsuo also conducted an extensive series of experiments to supplement his theoretical analysis. His is probably the only set of experiments to cover a very wide range of parameters - soil types, size, shape and depths of footings, vertical and inclined loads, and load controlled as well as displacement controlled type of testing. However, Matsuo also misses the point made by Turner regarding the maximum  $D/B$  ratio for anchors beyond which no significant increase in  $P$  is observed. As per Matsuo's formulae also,  $P$  goes on increasing with  $D$ ; though he recognises that the practical range of  $\frac{D}{B_1} = \lambda$  has an upper limit of 10, and divides the  $\lambda (= \frac{D}{B_1})$  ranges into three categories,  $0.5 \leq \lambda \leq 1.0$ ,  $1 \leq \lambda \leq 3$  and  $3 \leq \lambda \leq 10$ , where  $B_1 = \frac{B}{2}$  = half the footing width.

## 2.6. LATER WORK:

Alexander Vesic (18) has given results of his work on



the break-out resistance of objects embedded in ocean bottoms, an area where recorded laboratory or field data is quite scarce. Some of his interesting observations and conclusions are -

- (i) Very deep anchors under pull-out forces do not exhibit general shear failure, regardless of the relative density of the soil. Punching shear failure over a considerable vertical depth is observed. After being pulled upto relatively shallow depths, general shear failure pattern and a widening failure surface develop.
- (ii) Experiments on 3" plates give D/B ratios of about 2 and over 10 respectively for very loose and very dense sands, and about 2 and 5 respectively for very soft and very stiff clays, as CRITICAL RELATIVE DEPTH values, and below this ratio, i.e. at lesser depths, the behaviour of anchors is of shallow anchor pattern.
- (iii) Break out factors for deep anchors are practically equal to the corresponding bearing capacity factors of deep foundations.
- (iv) No equation can be fully satisfactory for all varieties of soil conditions.

Healy (2) reports experiments with smooth balls of wood and concrete 0.5" to 6.7" in diameter, pulled out by a

thin wire.  $P$ , the pull out force is related to the overburden pressure at the anchor depths, and is proportional to it, provided  $D > 6B$  for dense sands and  $D > 2B$  for loose sands. As per his tentative conclusions, the average pull-out stress is independent of anchor size in loose sands, and decreases in dense sands as anchor size increases.

## 2.7. THEORETICAL ANALYSIS:

Balla (1) and Matsuo (7,8) have rationally analysed to obtain theoretical values for the pull-out resistance. Since experimental values in the present investigation are compared with those computed from their two methods, these two analyses are presented here in a little more detail. For the sake of uniformity in comparison, the pull-out forces  $P$  expressed herein are excluding the self-weight of the footing.

The main step in the theoretical approach to the problem is to arrive at or assume the form of the sliding surface from the base of the footing to the ground surface. Balla assumes this as an arc of a circle rising vertically, i.e. tangentially to the horizontal base plate of the footing, and meeting the horizontal soil surface at an angle of  $(\frac{\pi}{4} - \frac{\phi}{2})$ , the angle which a failure plane in the Rankine passive failure zone makes with the horizontal. (Figure 2.4). Next the equilibrium of the footing and entire soil mass in the failure zone is



considered. Equating all the vertical forces to zero gives the pull-out force  $P$  in terms of the depth of embedment  $D$ , breadth of footing  $B$ , density  $\gamma$ , cohesion  $c$  and the friction angle  $\phi$  of the soil.

$$P = D^3 \gamma \left[ F_1 (\phi, \lambda) + F_2 (\phi, \lambda) + F_3 (\phi, \lambda) \right] \quad (\text{Eq. 2.1})$$

$$\text{where } \lambda = \frac{D}{B}.$$

$F_1 (\phi, \lambda)$ ,  $F_2 (\phi, \lambda)$  and  $F_3 (\phi, \lambda)$  are given in the form of charts over chosen ranges of  $\phi$  and  $\lambda$ .

Matsuo (7) assumes the sliding surface as a logarithmic spiral (Figure 2.5), starting from the edge 'c' of the footing, and meeting at 'd' the Rankine passive failure plane 'gd' through the surface point 'g' vertically above 'c'. From 'd', the surface is a straight line 'de' tangential to the log spiral at 'd'. The centre of the log spiral is a point 'O' on 'dg' produced. The particular logarithmic spiral and tangent straight line giving the failure surface is determined by trial and error. For each trial sliding surface such as 'cde', the equilibrium of the soil included in the portion 'abcdfa' is considered, taking into account -

- i) its own weight  $W$ ,
- ii) the friction  $F_1$  on the sliding surface,
- iii) the base pressure  $P_b$ , and

iv) the earth pressures  $P_1$  and  $P_2$  (the passive earth pressure) on the contact surfaces 'ab' and 'df'. The forces are considered separately for friction and cohesion cases, and that logarithmic spiral and tangent line are taken as the failure surface for which the pressure  $P_1$  ( $P_1 = P_{1f} + P_{1c}$ , where  $P_{1f}$  is the frictional component, and  $P_{1c}$  is the cohesion component) is a minimum.

In his next paper (8), Matsuo himself recognises that the procedure given by him for the exact evaluation of  $P$  involves tedious computations in spite of somewhat simplifying charts. He then makes a few further simplifications, as for example taking the angle  $\theta_0$  of the spiral as an average  $60^\circ$  for computation purposes, and finally arrives at the following expressions, which according to him give results close to the exact values.

$$P = \gamma B_2^3 K_1 + c B_2^2 K_2 - \gamma V_3 \quad (\text{Eq.2.2})$$

For different ranges of  $\lambda = \frac{D}{B_1} = \frac{D}{B/2}$ ,

$$0.5 \leq \lambda \leq 1,$$

$$B_2^3 K_1 = (0.056 \phi + 4.000) B_1^3 \cdot \lambda^{(0.007 \phi + 1.000)}$$

$$B_2^2 K_2 = (0.027 \phi + 7.653) B_1^2 \cdot \lambda^{(0.002 \phi + 1.052)}$$

$$1 \leq \lambda \leq 3,$$

$$B_2^3 K_1 = (0.056 \phi + 4.000) B_1^3 \lambda^{(0.016 \phi + 1.100)}$$

$$B_2^2 K_2 = (0.027 \phi + 7.653) B_1^2 \lambda^{(0.004 \phi + 1.103)}$$

and for  $3 \leq \lambda \leq 10$ ,

$$B_2^3 K_1 = (0.597 \phi + 10.40) B_1^3 \left(\frac{\lambda}{3}\right)^{(0.023 \phi + 1.300)}$$

$$B_2^2 K_2 = (0.013 \phi + 6.110) B_1^2 \lambda^{(0.005 \phi + 1.334)}$$

(Eqs. 2.3)

where  $B_1$  = Half the base width of footing,

and  $\phi$  = Friction angle.

For the case  $c = 0$ , the pull-out force (Eq. 2.2) can be written as,

$$P = m \cdot B_1^3 (n \lambda)^l \quad (\text{Eq. 2.4})$$

with appropriate expressions for  $m$ ,  $n$ ,  $\lambda$  and  $l$  for the different ranges from Eqs. 2.3.

## 2.8 GENERAL COMMENTS:

The earlier methods have been reviewed and some of their inherent discrepancies have been noted. The values given by Balla and Matsuo seem to be based on rational principles in spite of the fact that the slip surfaces have been assumed in their analyses.

## CHAPTER 3

### THE EXPERIMENTAL STUDIES

#### 3.1 INTRODUCTION:

Experiments are conducted for the pull-out resistances in cohesionless soils. The details are given below.

#### 3.2. THE SOIL:

The granular soil used is the sand from the river Ganges near Kalpi. All experiments have been conducted with the soil in the air-dry state. The soil is of the fine variety of sand, with a low range of densities. In the loosest state, it weighs 1.40 g/cc. When compacted under saturated conditions, its maximum dry density is found to be 1.725 g/cc. However, in the air-dry condition, when compacted in the Proctor's mould in 3 layers with 25 blows per layer of the 5.5 lb. hammer, the density is 1.65 g/cc and probably this is about the maximum density we can aim at, for air-dry sand. The specific gravity  $G$  for the soil solids is 2.57.

In the loosest state, the angle of internal friction  $\phi$  is found to be  $27^\circ$ , and in the densest state  $\phi$  is  $36^\circ$ . Between these two values a straight line variation for  $\phi$  is assumed (after Scott) (12), (Figure 3.1). The cohesion  $c$  is practically zero.

The grain size analysis curve (figure 3.2) indicates the sand is substantially uniform.

### 3.3. THE SOIL BED AND DENSIFICATION:

It is proposed to use fairly large size models, and depths of upto 60 to 70 cm. A large size wooden tank (figure 3.3) of approximately 1.5m x 1.5m x 1.0m is made of 2.5cm thickness throughout. At a later stage, a second tank, 0.9m x 0.75m x 0.65m, also of wood, is used in addition for conducting tests on smaller models, as also for using a soil bed in dense state.

Sand densification involves three stages, viz.,

1. Method of compaction,
2. Achieving uniformity of density through the soil bed, and
3. Measurement of the density attained.

Ramming, rolling, vibration and inundation by flooding the bed have been the common methods of sand compaction. Of these, the last mentioned is unsuitable to laboratory work, particularly when the idea is to work on air-dry sand, because it is always difficult to verify if the lower layers have become completely dry, even after drainage. Rolling is a method more suited to field compaction. Thus ramming and vibration have been the usual laboratory methods, of which the latter is chosen for this work. For the initial stages



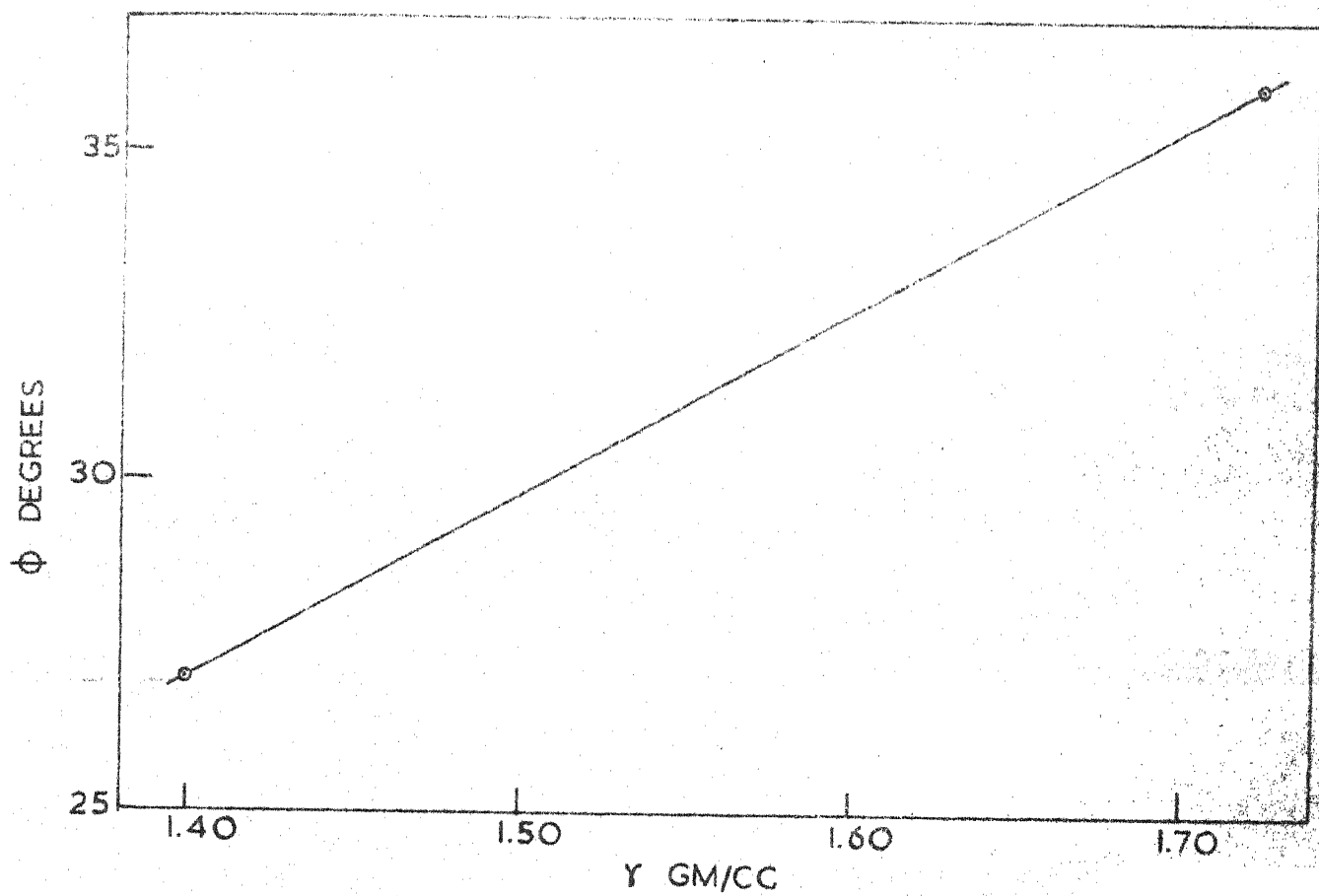


FIG. 3.1\_VARIATION OF  $\phi$  WITH DENSITY

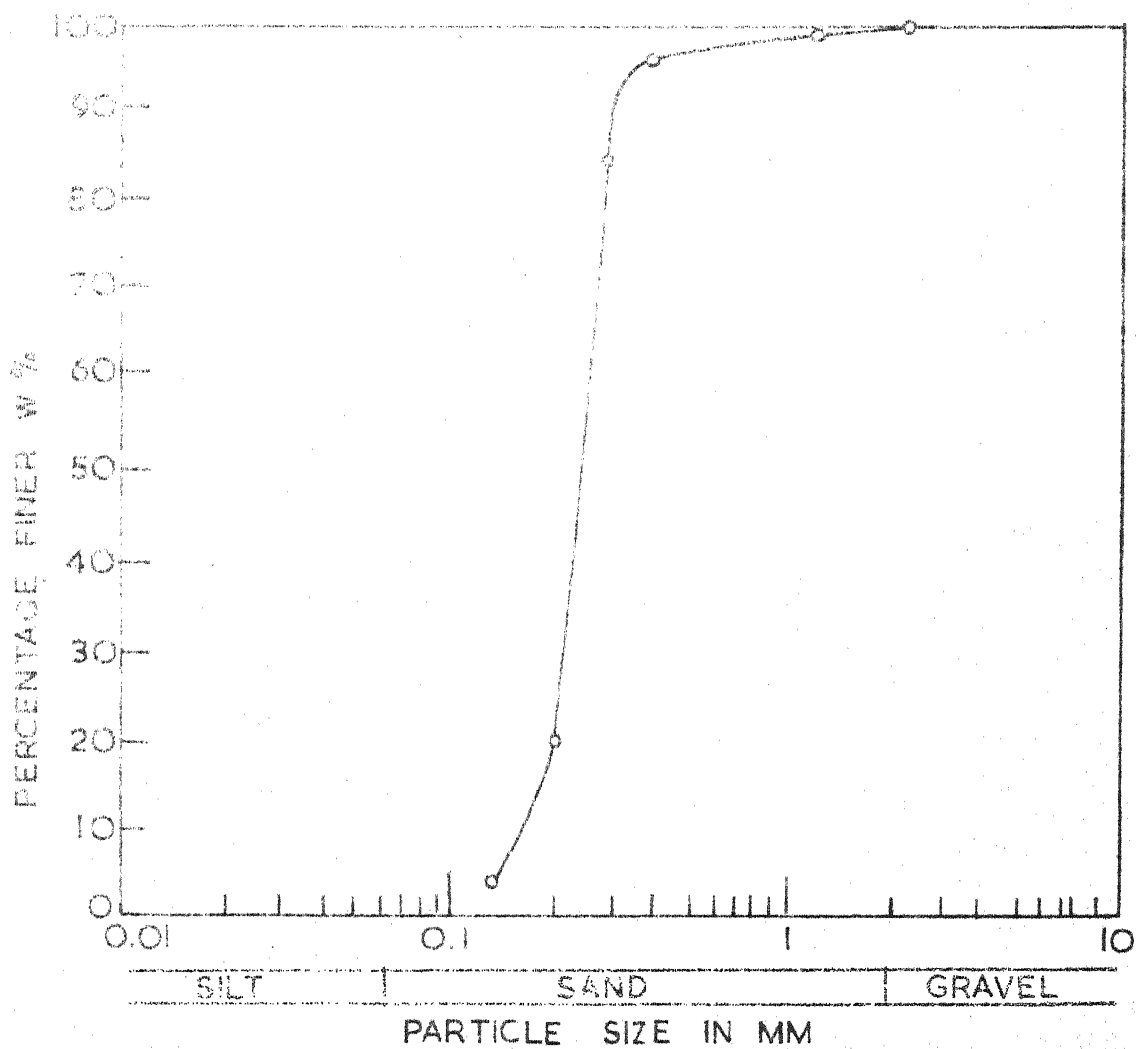


FIG. 3.2 \_GRAIN SIZE DISTRIBUTION OF SAND

of the experiments, vibration is done with an immersion or poker type of vibrator (Type SC 8976, 0.5 H.P., PH1, Cycles 50, V. 230, RPM 1500), which is put in 30 cm layers of sand at intervals of 30 cm in both directions. Even with this close spacing, as observed, the density attained is around 1.5 g/c.c. and sometimes even less. Increasing the time of vibration at each point is not effective beyond 45 seconds or 1 minute. The variation in density at different points is usually  $\pm 0.02$  and occasionally 0.03 g/c.c. of the average, and this is treated as fairly uniform.

It is felt that one method to make vibration more effective is to vibrate the entire box as a whole along with the sand and the embedded model. The first tank is too heavy for this within the limitations of equipment. The second is then selected for this purpose. This tank is supported on springs, reinforced with angles and tie rods in the direction of the vibrator force, and the vibrator, fixed on the top at the centre of the box between supporting angles, is switched on. It is found that the observed density reached each time is at or near the maximum that could be expected for air-dry sand, i.e. 1.65 g/c.c.

Attaining uniform density and the measurement of that density are important aspects of sand densification, even more than the method of compaction itself. The common method

of measuring the density of sandy soils has been to place a number of small containers at horizontal and vertical intervals, and measuring their volumes and weights of contained soils after their removal. The sand cone method of ascertaining in-situ densities is not suited to these beds, for three reasons: firstly, any dense sand, scooped out from the bed, leaves the pit which does not retain its shape, the surrounding soil becomes loose, and the volumes measured are not the true volumes of the dense sand; secondly, sands, however dense inside, are relatively loose for small depths at the surface; and thirdly, the method is suited to find surface densities only

During the present investigations also containers have been kept at the centres of 30 cm squares horizontally in layers at a vertical interval of 30 cm. There are no means to verify, in the case of the large tank, if the densities given by these containers are correct. They are just assumed as representing the in-situ density of the sand-bed.

However the same procedure has been repeated with the small tank, which is vibrated as a whole with all its contents. The height of the same volume of sand is observed before and after vibration. When sand is just filled into a Proctor's mould, the average of 3 readings gives the uncompacted sand density as 1.42 g/c.c. If filling sand inside the box by unskilled labour either with a little force or from a little

height has any effect, it can only slightly increase this by + 0.01 or 0.02 g/c.c. However, even with an uncompacted density taken as 1.42 g/c.c., the computed density based on the heights of sand inside the box before and after vibration, is 1.65 to 1.67 g/c.c., whereas that indicated by the containers is 1.50 to 1.54 g/c.c. except when the containers are near the surface, say 15 to 20 cm from it, in which case it has gone upto 1.57 g/c.c., and only one container in one test has recorded 1.60 g/c.c.. Changing the size, and/or the diameter - height ratio of containers have shown no better results. It is thus evident that the container method of ascertaining densities is not really representative of the soil densities. It is observed however, that the variation in container densities is not much, and is usually  $\pm 0.02$  g/c.c. of the average and occasionally (about 1 in 10 or 12)  $\pm 0.03$  g/c.c. With the observation just made, probably this can be taken at least as indicative of the state of uniformity in the density of the bed.

During the present investigations, it is assumed that with this range of variation in the container densities, the soil can be treated as approximately or practically uniform, while the average density is calculated on the basis of an uncompacted density of sand of 1.42 g/c.c. and the depths of sand in the tank before and after vibration.

While it is realised that the aforementioned observations regarding the sand density values given by the container method are valid strictly for this vibration method of compaction followed in these tests, it is evident that the effect will probably be the same even in the case of ramming, unless heavy ramming AT THE SPOT WHERE A CONTAINER IS POSITIONED forces more soil directly below into the container; in which case, it is clear that the container represents the density at the spot, and not necessarily of the soil in general.

#### 3.4. THE EXPERIMENTAL SET-UP:

The set-up is shown in figure 3.3. The footing model is placed over a bed of sand, centred to position, and further sand filled up to the required depth of embedment. A beam resting on two columns on either side of the tank forms the support on which the loading jack rests. A tension hanger being pushed up by the jack transfers the pull through a central wire rope looped on to a hook formed in the shaft of the footing. A proving ring placed between the jack and the hanger measures the tensile force on the model. A steel ball between the proving ring and the hanger minimises friction and keeps the load vertical.

The second set-up (figure 3.4) is similar but smaller in dimensions and is used for smaller models, depths upto

40 to 45 cm and dense sand. The same jack and proving ring and similar hanger is used here also. The tank is reinforced with angles and tie rods, and to enable it to be vibrated as a whole along with the contents (sand bed and embedded model) it is rested on 8 springs which allow it to be shaken freely.

### 3.5. THE FOUNDATION MODELS:

10 models have been tested in all at various depths, 4 with the sand bed at densities of 1.5 g/c.c. and the others at 1.65 g/c.c. Table 3.1 shows the list of 4 circular, 2 square and 2 rectangular models. In addition, a square and a circular model among the above are modified with 4 projections added to each. The idea is to test if such small increases in footing areas add substantially to the pull-out resistance.

### 3.6. THE EXPERIMENTS:

#### 3.6.1. PULL-OUT FORCE:

In all 34 tests have been conducted on the 10 models, For the tests in the large tank, sand is filled to the required depths in layers of not more than 30 cm. and the immersion vibrator is used as described earlier. The top layer of a few inches of sand in every case is made up by hand as the vibrator left depressions at the points of immersion.

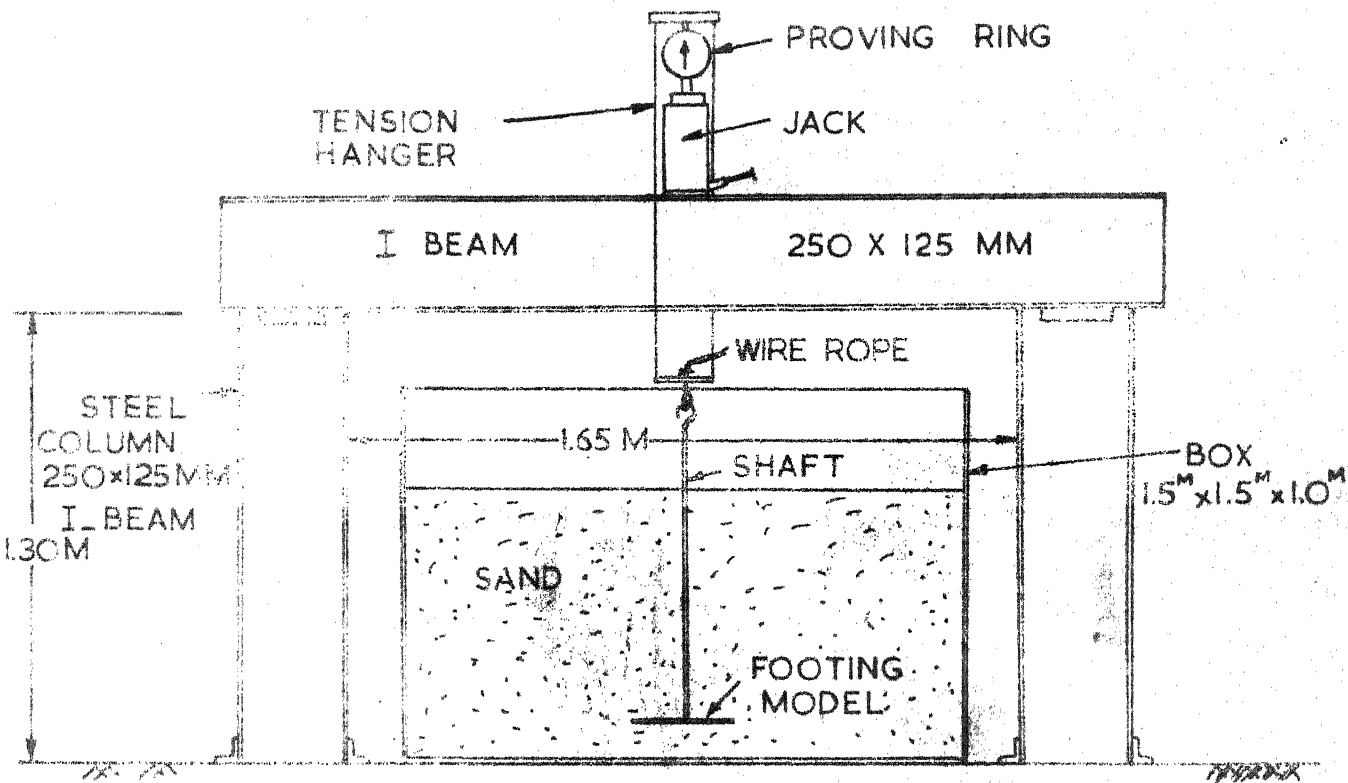


FIG. 3.3 EXPERIMENTAL SET-UP 1



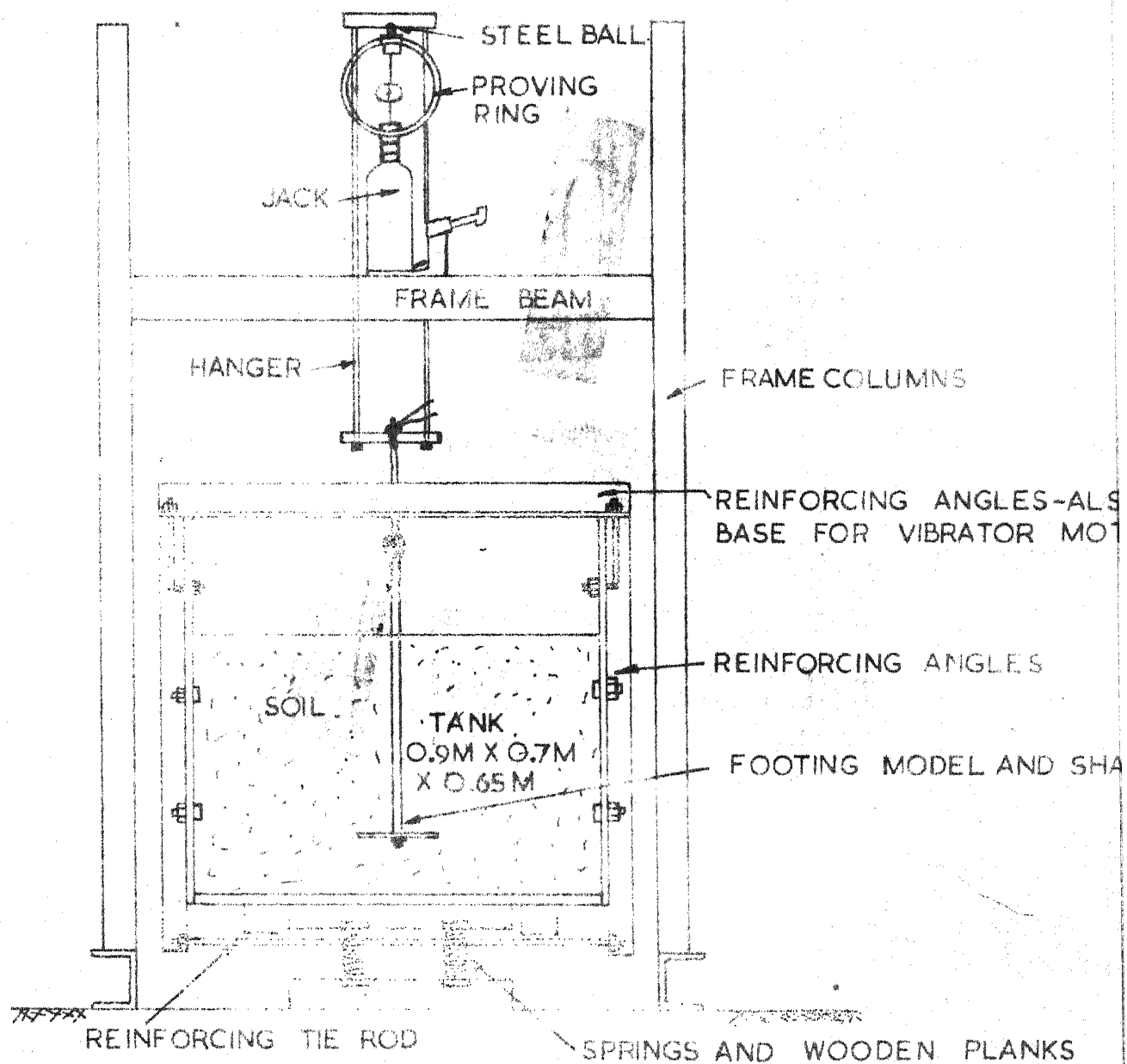


FIG. 3.4 \_ EXPERIMENTAL SET-UP \_ 2

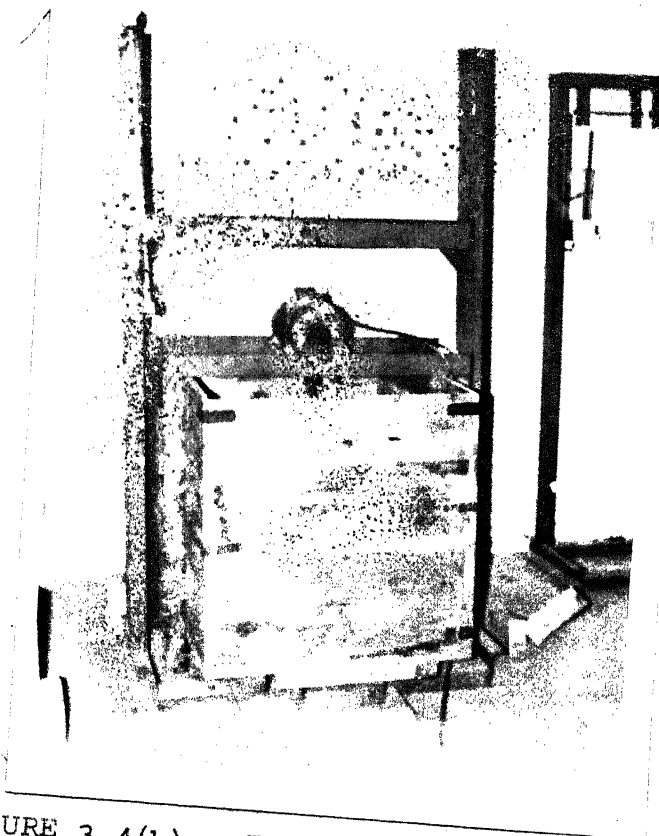


FIGURE 3.4(b) : EXPERIMENTAL SET\_UP 2

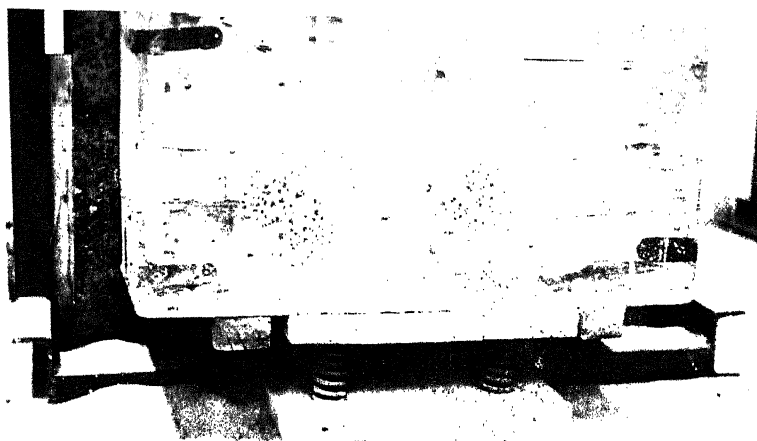
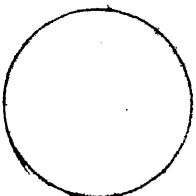


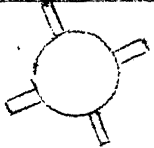
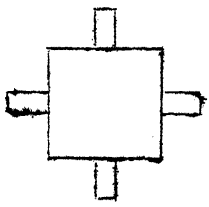


FIGURE 3.4(c) : EXPERIMENTAL SET\_UP 2



FIGURE 3.5 : SOME OF THE FOUNDATION MODELS

TABLE 3.1 : LIST OF MODELS TESTED

Shape	Model No.	Size cm	Equivalent B in Balla's eqn. cm	A cm <sup>2</sup>	p cm
 Circle	1	Dia. = 37.5	37.5	1100	117.8
	2	30.0	30.0	707	94.2
	3	20.0	20.0	314	62.8
	4	15.0	15.0	177	47.1
 Square	5	Side 20.0	22.6	400	80.0
	6	15.0	16.9	225	60.0
 Rectangle	7	20.0 x 40.0	32.0	800	120.0
	8	20.0 x 30.0	27.7	600	100.0
	9	15 cm. dia. circle with 4 projections each 2 cm wide x 6 cm long	16.9	225	95.1
	10	15 cm square with 4 nos. projections each 2 cm wide, 5 cm long.	18.3	265	100

The axial load is applied on the footing by the hand operated jack. Beyond the maximum load, the displacements are large even at smaller loads.

For tests in the small tank, which always rested on a bed of springs, the footing is centred after a layer of sand is first laid, and the sand filled up. The eccentric motor is fixed to angles resting in turn on the reinforcing angles at the sides of the box, and the sand is compacted. The motor is removed, and the loading jack connected to the shaft. Pairs of wooden wedge pieces driven tight on either side between the sides of the box, and the loading frame, prevent tilting of the tank, which might otherwise be possible as the same is resting on springs. The rest of the experiment is same as in the earlier set up.

Tables 3.2 and 3.3 give the results of 33 tests. Listed in these tables are values of pull-out forces  $P$  obtained in the tests, as also the values calculated from Balla's charts and Matsuo's expressions (Section 2.7). For footings of shapes other than circular, the latter computations are based on an equivalent  $B$  = the diameter of a circle of equal area. Figures 3.6 through 3.15 graphically represent the same results.

### 3.6.2: LOAD Vs. DISPLACEMENT:

During the experiments on the 15 cm circular and 15 cm square models, the displacements of the footings are also

recorded with the help of a dial gauge. The load-displacement plots are shown in figures 3.16 and 3.17.

### 3.6.3.: AVERAGE PULL OUT FORCE PER UNIT AREA:

Figures 3.18 and 3.19 give the average force  $\frac{P}{A}$  ( P = Pull out force; A = Area of footing) vis-a-vis the D/B ratios for all the tests conducted, in the relatively loose and the dense sands.

### 3.6.4.: RUPTURE SURFACE:

In air-dry sand, it is very difficult to observe the rupture surface, and more so in the fine variety as in these areas. Even at the top surface of the soil, the rupture line is hardly or not at all visible at the point of maximum load. On pulling out a little more, probably some of the sand which obviously loosened on the rupture surface falls off and gives rise to a new rupture surface. This presumption is based on the observation that on pulling out a little more, till a clear line of rupture is observed at top, it is found that this line is much inside the zone where surface heaving of sand was observed earlier.

To make at least a qualitative study, a pull out test is conducted by placing layers of sand in 7 to 8 cm thicknesses with thin layers of white coloured cement in between. Lest

vibration may disturb the whole layers, each layer is rammed by hand before the cement is spread on it. After the required depth is laid, water is carefully poured, soaking all the layers. The pull out test is immediately performed to a displacement of over 2 to 3 cm. (Even so, the fine sand closes the gaps very quickly). A time gap is allowed for the cement to set. The sand is later on scooped out carefully, first along two radial planes at the boundaries of a quadrant, and later in another quadrant. For the remaining semi-circle, sand is removed layer by layer. The outermost observed failure surface has again been not clear, but could be made out with some difficulty, and is shown plotted in figure 3.20.

TABLE 3.2 : PULL OUT FORCES ON FOOTING MODELS-I

 $(\gamma = 1.5 \text{ g/c.c.})$ 

Test no.	Model No.	Equivalent B cm	Depth D cm	PULL OUT FORCE IN Kg.		
				TEST	BALLA	MATSUO
1	1	37.5	17.5	43.5	22.5	52.5
2			37	128.5	178	163.5
3			57	343	389	282.5
4	2	30.0	15.5	26	14	29.5
5			30	66.5	91.5	84
6			42.5	128	176	152.5
7			57.5	270	324	227
8	7	32.0	19.6	44	23	45.5
9			36.4	118	130	120.5
10			55.5	276	320	229
11	8	27.7	21.2	35	25	44
12			32.5	79	91.5	85.5
13			57.5	282	306	208



TABLE 3.3 : PULL OUT FORCES ON FOOTING MODELS-II

$$\gamma = 1.65 \text{ g/c.c.}$$

Test no.	Model no.	Equiv- lent B cm	Depth D cm	PULL Out Force in Kg.		
				TEST	BALLA	MATSUO
14	2	30.0	29.5	129.5	101	100
15	3	20.0	22.5	45	37	37
16			30.5	100.5	68.5	51.5
17			40.5	202	125	90.5
18	4	15.0	15.5	18.5	14	13.5
19			29.5	79.5	50	36.5
20			46.2	214	140	97
21	5	22.6	21.4	55	44	41.5
22			31	129	81	75.5
23			41.4	234	147	108
24	6	16.9	15	19.5	15	15
25			33.5	110.5	72.5	55
26			45.5	285.5	140	104.5
27	9	16.9	15.3	24	9	15
28			22.5	46	30.5	27.5
29			30.8	102.5	60	44
30			46	275	146	101.5
31	10	18.3	21.5	41.5	29	29.5
32			33	131.5	75.5	51
33			43	234	135.5	95

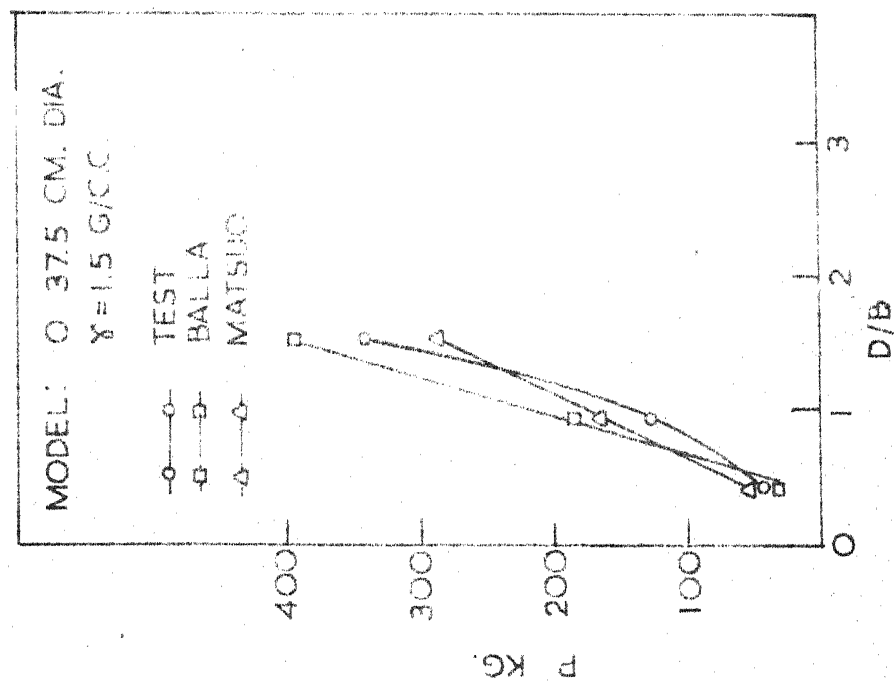


FIG.3.6\_PULL-OUT FORCES FOR  
 MODEL-1

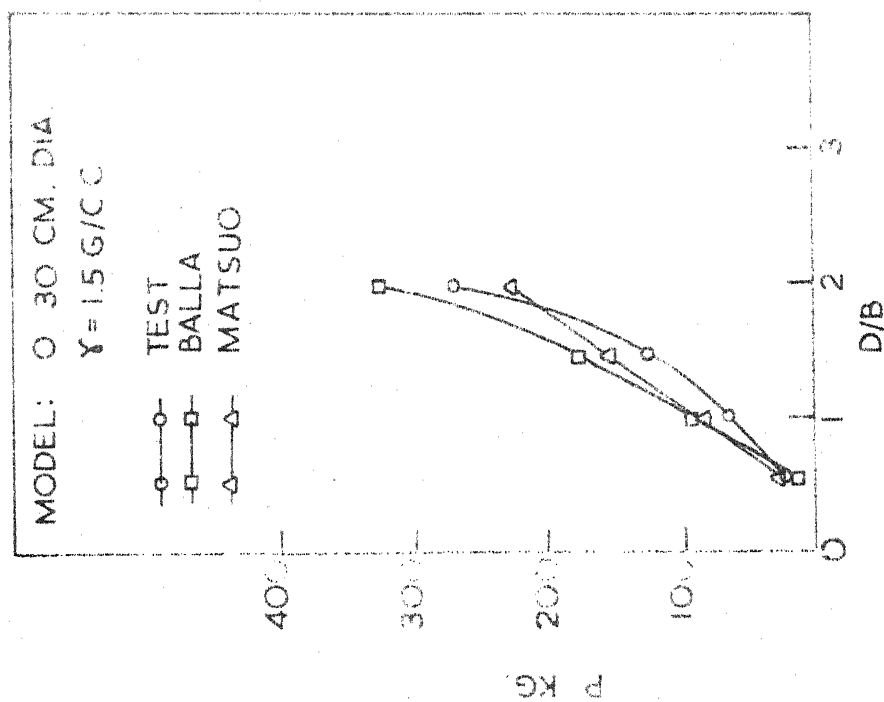


FIG.3.7\_PULL-OUT FORCES FOR  
 MODEL-2

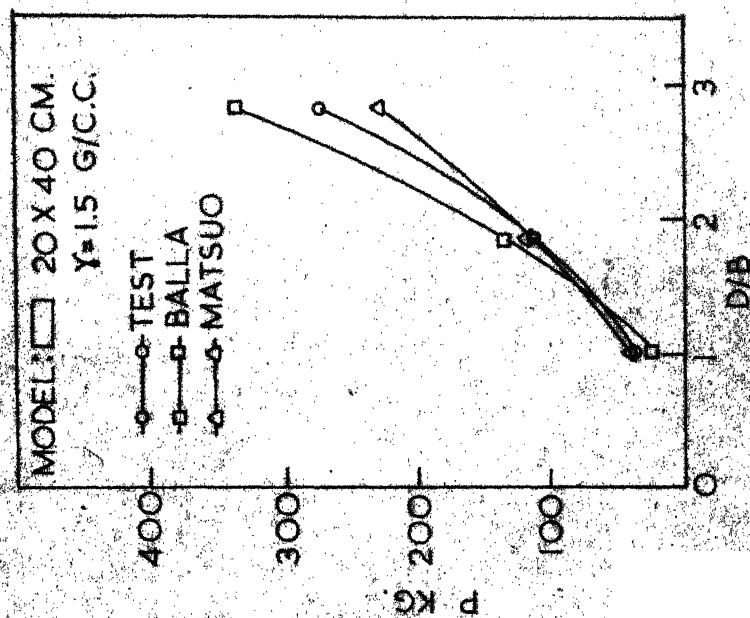


FIG. 3.8 - PULL-OUT FORCES  
 FOR MODEL 7

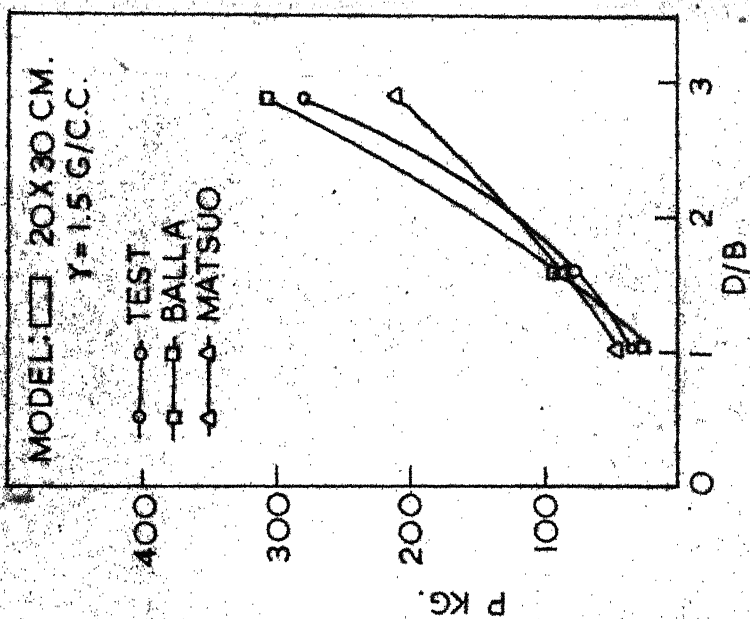


FIG. 3.9 - PULL-OUT FORCES  
 FOR MODEL 8.

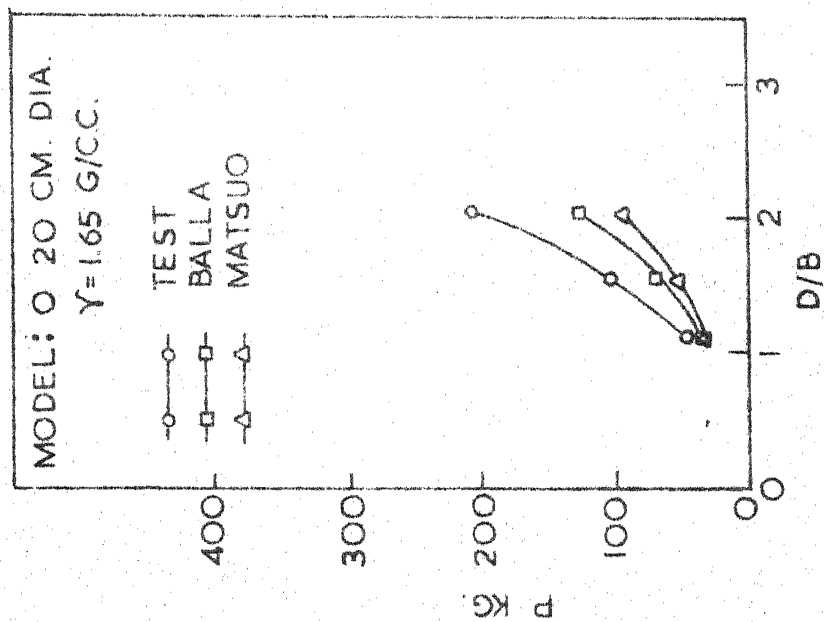


FIG.3.10\_PULL-OUT FORCES  
 FOR MODEL 3

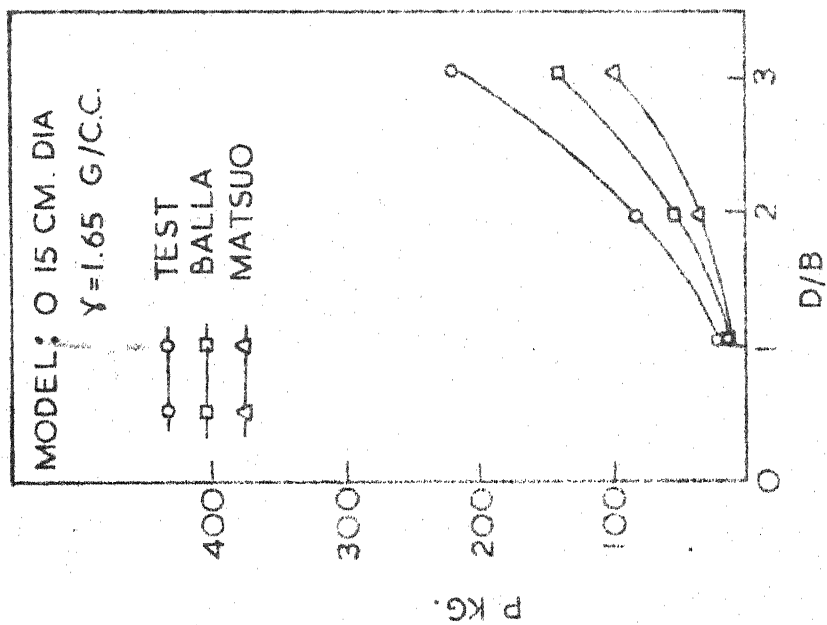


FIG.3.11\_PULL-OUT FORCES  
 FOR MODEL 4

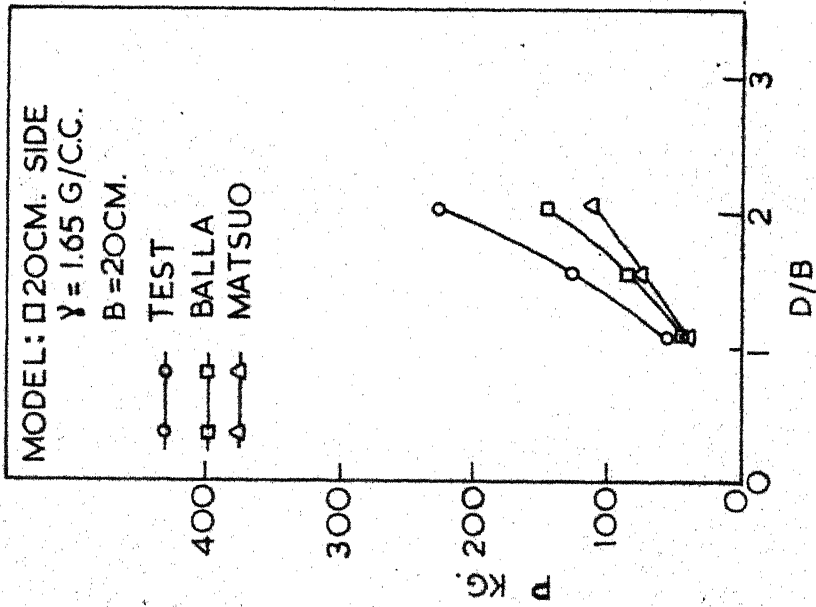


FIG.3.12\_PULL-OUT FORCES  
FOR MODEL\_5

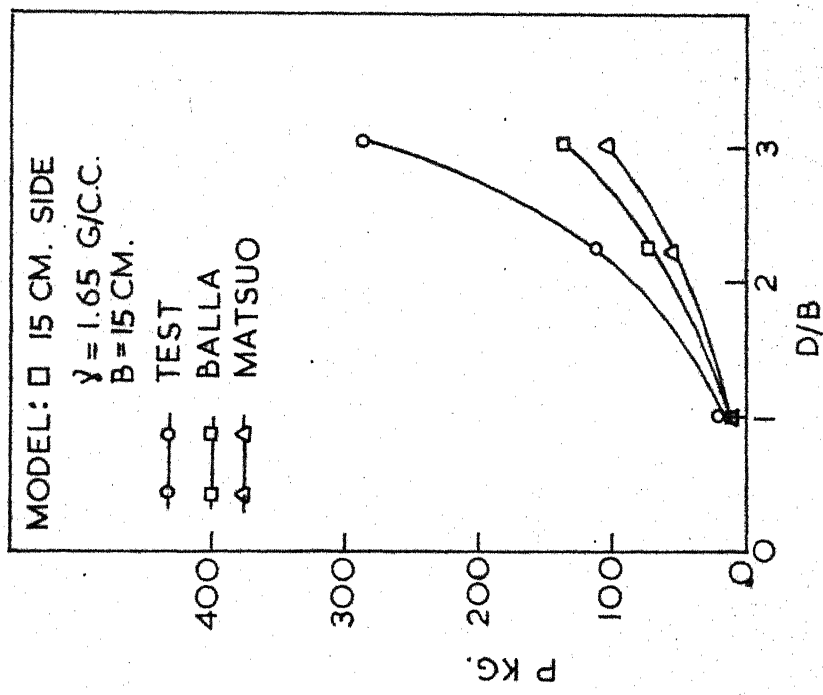


FIG.3.13\_PULL-OUT FORCES FOR  
MODEL\_6

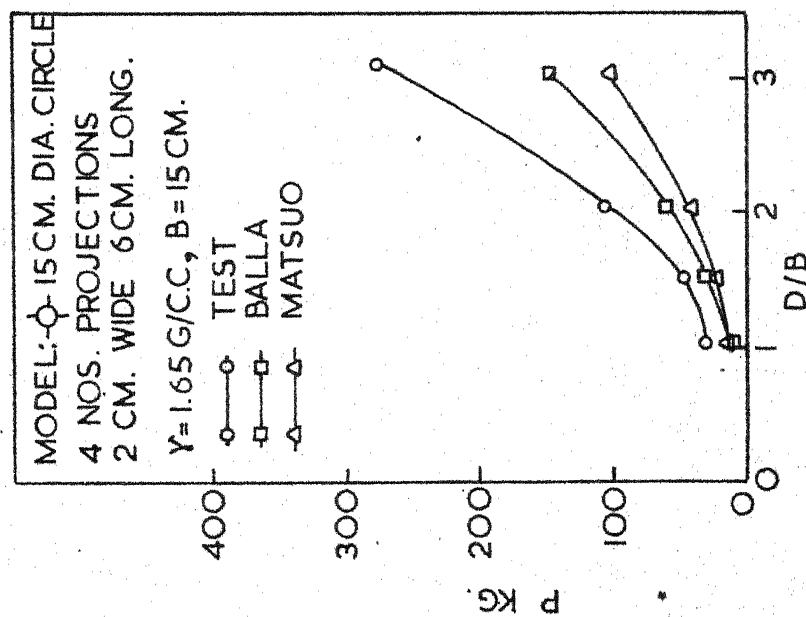


FIG. 3.14. PULL-OUT FORCES  
 FOR MODEL -9

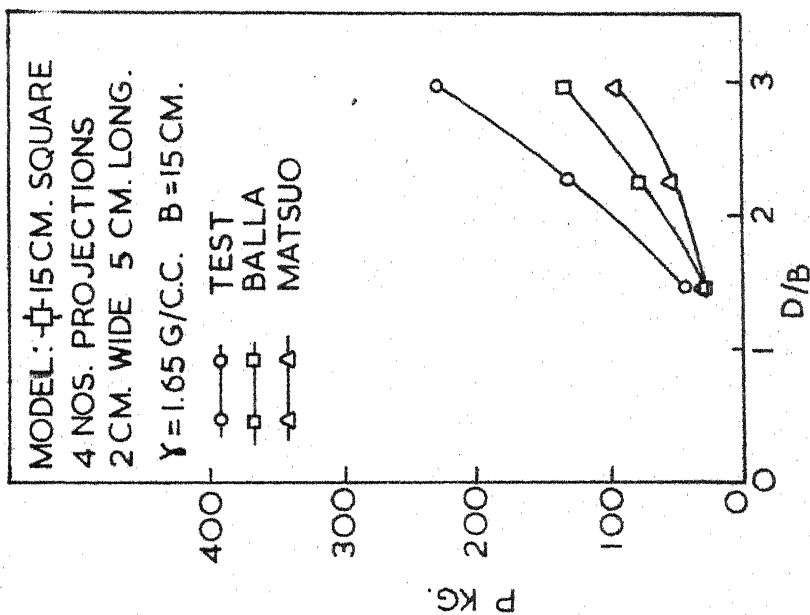


FIG. 3.15. PULL-OUT FORCES  
 FOR MODEL 10

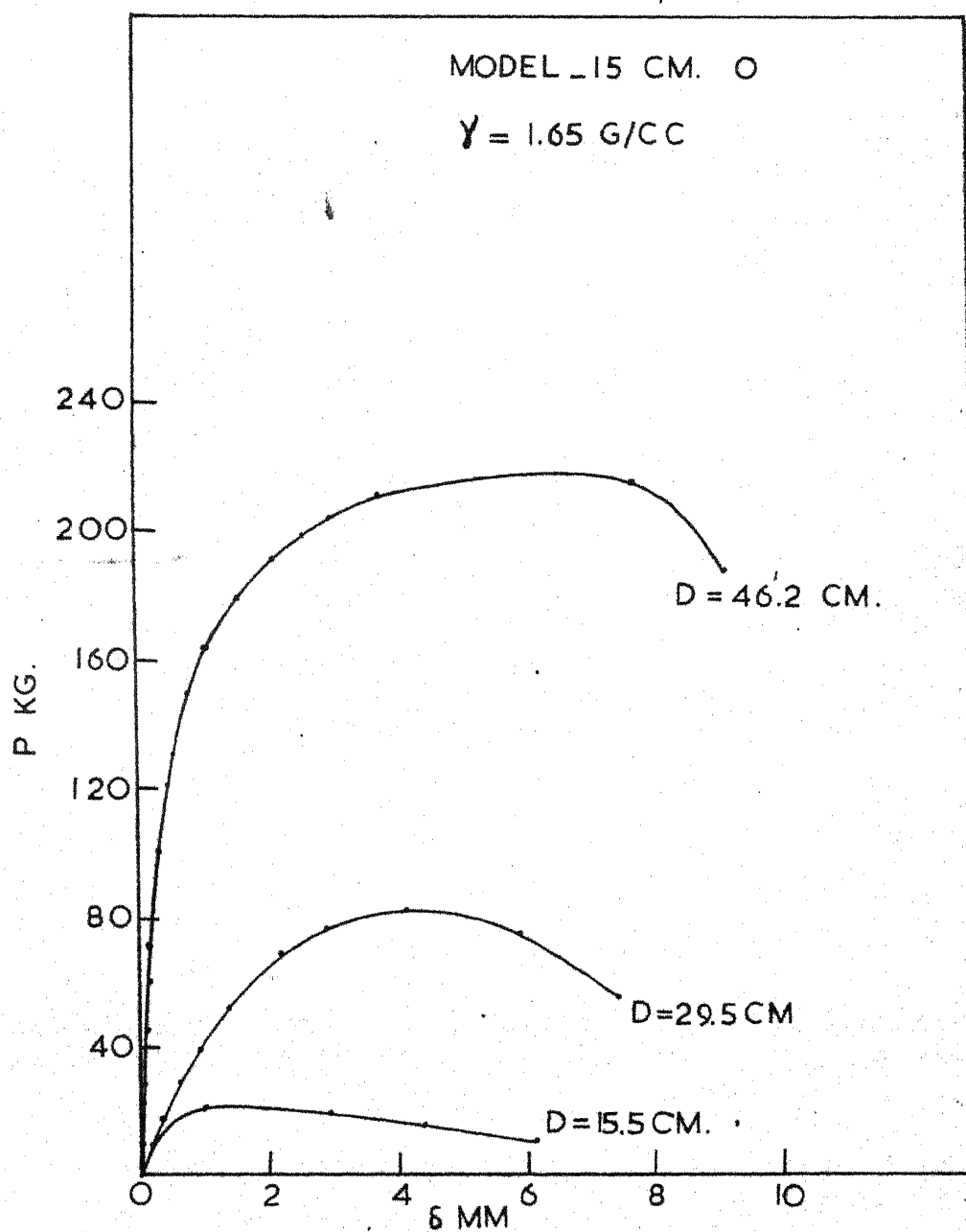


FIG. 3.16\_LOAD - DISPLACEMENT CURVES FOR -  
MODEL \_4

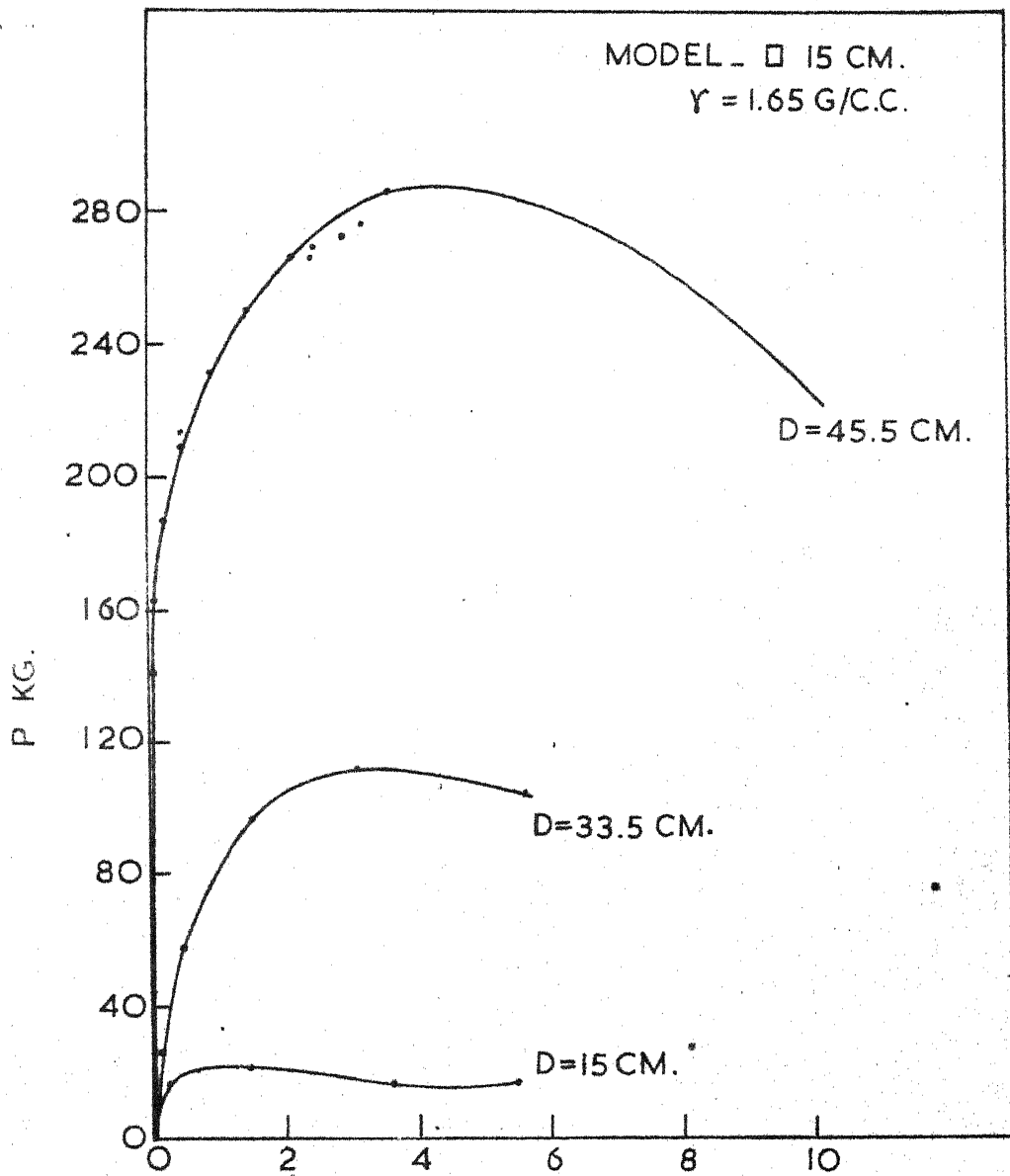


FIG.3.17\_LOAD - DISPLACEMENT CURVES FOR  
MODEL - 6



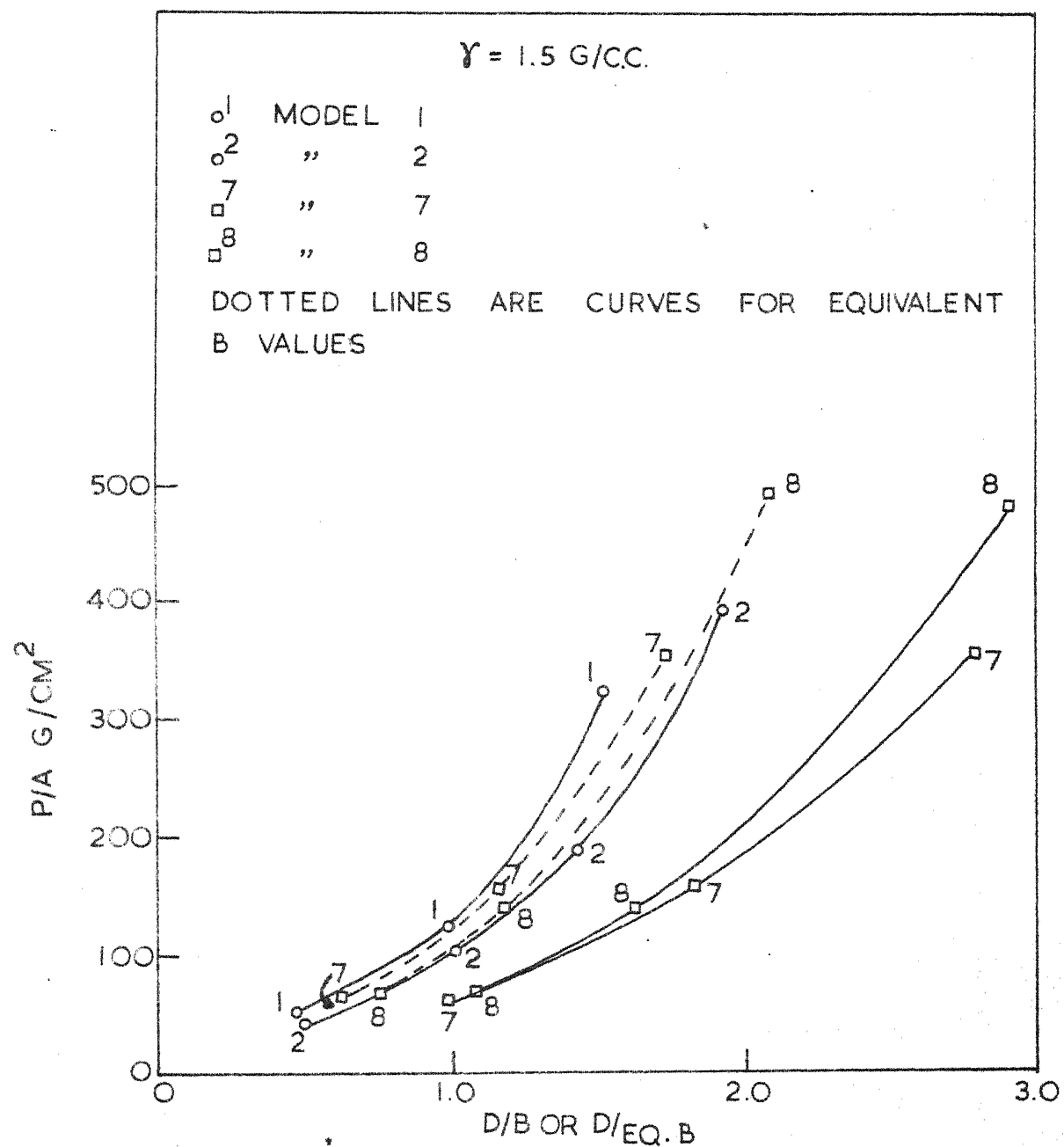


FIG. 3.18\_AVERAGE PULL-OUT PRESSURES IN RELATIVELY  
LOOSE SAND

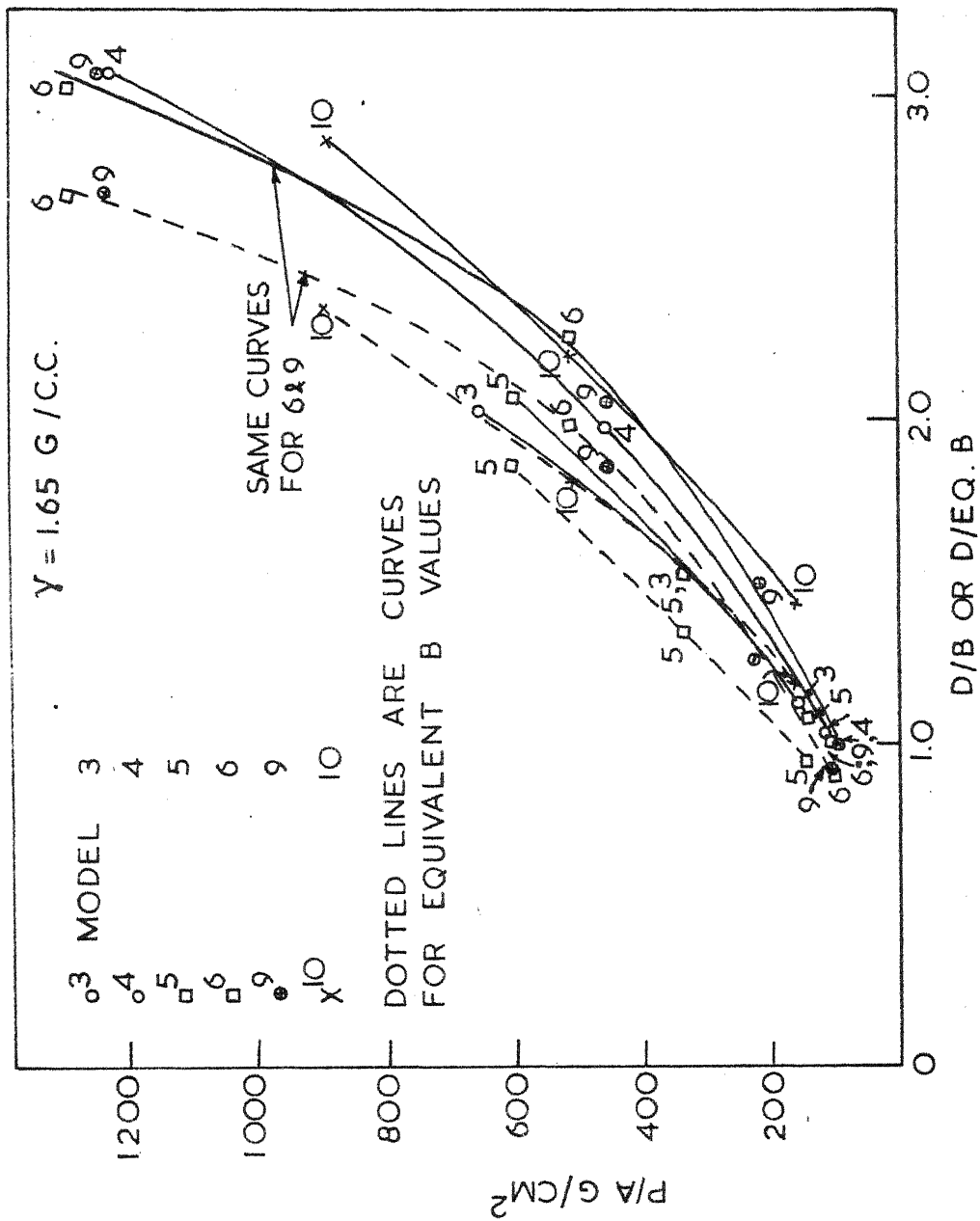


FIG. 3.19\_AVERAGE PULL-OUT PRESSURES IN DENSE SAND.

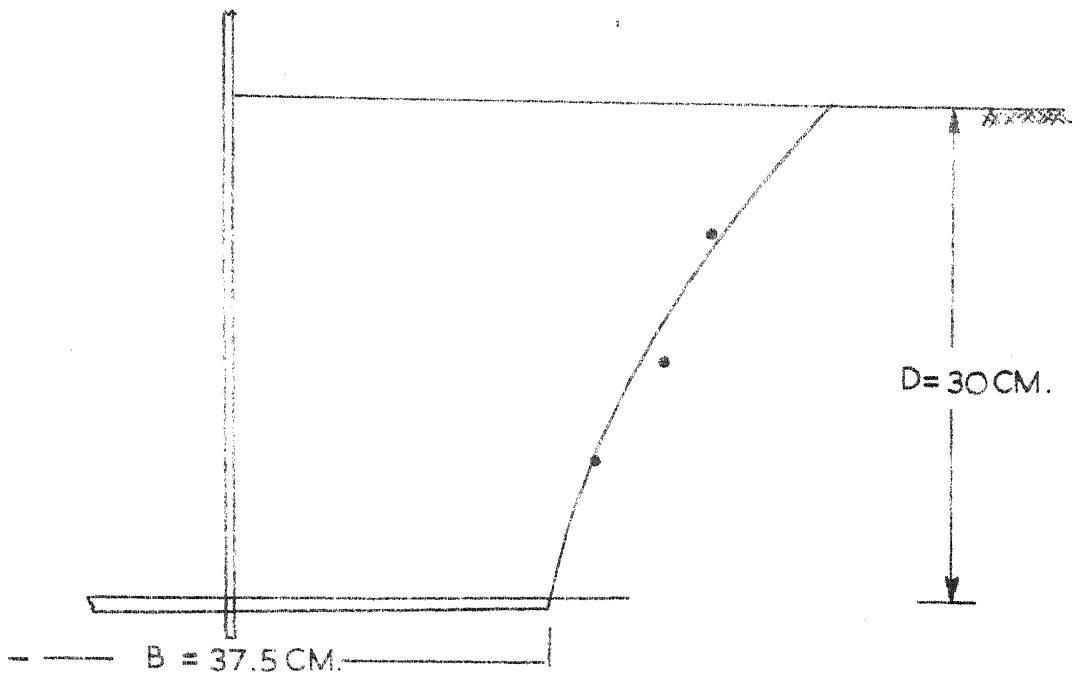


FIG. 3.20. RUPTURE SURFACE

## CHAPTER 4

### RESULTS AND COMPARISONS

#### 4.1. COMPARISON OF ULTIMATE PULL OUT FORCES:

Figures 3.6 through 3.9 show the results of pull out resistance versus D/B values, for relatively loose sand. The plots show that Balla's values give lower values than tests for shallow footings and high values for higher D/B ratios. Even for D/B of 1.5 to 2.0 Balla's values are higher by as much as 15 to 20 percent of test results. Matsuo's values are generally lower than given by tests, though over a small range, the reverse is the case.

However, both Balla's and Matsuo's expressions can be seen to underestimate the pull out resistance in fine grained dense sand (Figures 3.10 through 3.15). Test values are higher than Matsuo's by over 100 percent in some cases, Balla's accuracy is only slightly better. Thus both the theories appear to be inadequate for this case.

#### 4.2. EFFECT OF FOOTING SHAPE:

Figures 4.1 and 4.2 give the test values of P for the various models at different depths. In figure 4.3 is plotted the Load ratio = Load taken by any model/load taken by Model 2 at the same depth for ( $\gamma = 1.5$  g/c.c., vide

Table 3.2, and Load ratio = Load taken by any model/Load taken by Model 4 at the same depth, for  $\gamma = 1.65$  g/c.c. The cost of footing material, as also a major portion of excavation is proportional to the area of footing, whereas cost of temporary retaining structures, if necessary, depends on the perimeter. The area ratios  $A_r$ , and perimeter ratios  $P_r$  are also given, taking the values for models 2 and 4 as unity in each case.

The following inferences can be drawn from the above figures:

(i) It is proposed to find out whether significant increase in pull out capacity is obtained by using projections like those in models 9 and 10. It is seen that models 6 and 9, both having the same area, have about the same P vs. D curve (Figure 4.1). This means no particular advantage is derived by going in for such shapes. On the other hand, it is costly too.

(ii) For a given depth of footing, in relatively loose or medium sand, a rectangular footing appears to have greater pull out resistance in the range of depths tested, say  $\frac{D}{B} < 3$ . Even here, footing with the sides ratio  $\frac{L}{B} = 1.5$  shows a better performance than the one with the ratio of 2. The load ratio of this model is consistently higher; i.e. for

given ratio in area of the model to that of a circular footing, the pull out force is more than proportionately greater than that of the same circular footing. In contrast, increasing the diameter of the circular footing (Model 1 vs Model 2) has not shown a corresponding increase in P values.

(iii) In dense sand, however, in the range tested, the smaller circular footing is about the best choice. A square footing has not shown a proportionate increase in P, but the trend of the curve appears to indicate that this may be the case at still greater depths. This however needs verification.

#### 4.3. A NON-DIMENSIONAL PLOT:

An interesting feature is observed in the non-dimensional plot of figure 4.4, in which the ratios of  $D/B$  are taken as abscissa. B is the diameter of the circular models, and for other shapes, it is the diameter of a circle of equal area. The ordinates are the ratios of average ultimate pull out stress  $q$  per unit area of footing ( $q = \frac{P}{A}$ ; A = Area of footing) to the over-burden pressure  $q_0$  at the footing level ( $q_0 = \gamma D$ ;  $\gamma$  = the average density of the soil, and D = Depth to the top of the footing). For each density (in this case there are two), an average common line can be drawn through all test data in that group. The dotted lines show the lower bounds of the values.

Before any conclusions can be drawn, it will be necessary to conduct series of tests at more density values. However, if the trends observed here are any indication, it should be possible to arrive at simple design equations, for the usual shapes of circular, square and rectangular ( $L/B = 1$  to  $2$ ) footings. Either the average curve or even the lower bound curve can be used with a change in the Safety Factor.

#### 4.4. SHAPE OF RUPTURE SURFACE:

The soil material in the qualitative test (section 3.6.4) is somewhat different in that cement layers are spread between sand layers, and also water is present. The effect of this difference is not known. As observed however, the rupture surface appears to be a logarithmic spiral extending right from the base of the footing to the surface (Fig. 3.20).

#### 4.5. EFFECT OF DENSIFICATION OF SAND:

Tests numbered 5 and 14 (Tables 3.2 and 3.3) give an indication of the increase in the pull out capacity due to densification of sand. For the same model 2, at density  $\gamma = 1.50$  g/c.c.,  $P$  is found to be 74 kg at a depth of embedment of 30 cm ( $D/B = 1.0$ ), whereas in the dense stage ( $\gamma = 1.65$  g/c.c.), it has gone up to 137 kg or by nearly 90%.

#### 4.6. LOAD-DISPLACEMENT (P- $\delta$ ) CURVES:

Figures 3.16 and 3.17 give the curves for different embedment depths in the case of model no. 4 (15 cm circle) and model no.6 (15 cm square). In each case, the P- $\delta$  behaviour is linear upto about  $\frac{1}{2}$  to  $\frac{2}{3}$  the ultimate load, but the Modulus is seen to increase with the depth of embedment, in the range tested. This may probably mean that a minimum depth of embedment is necessary for each size of footing if the elastic properties of the soil are to be utilised to a greater extent.



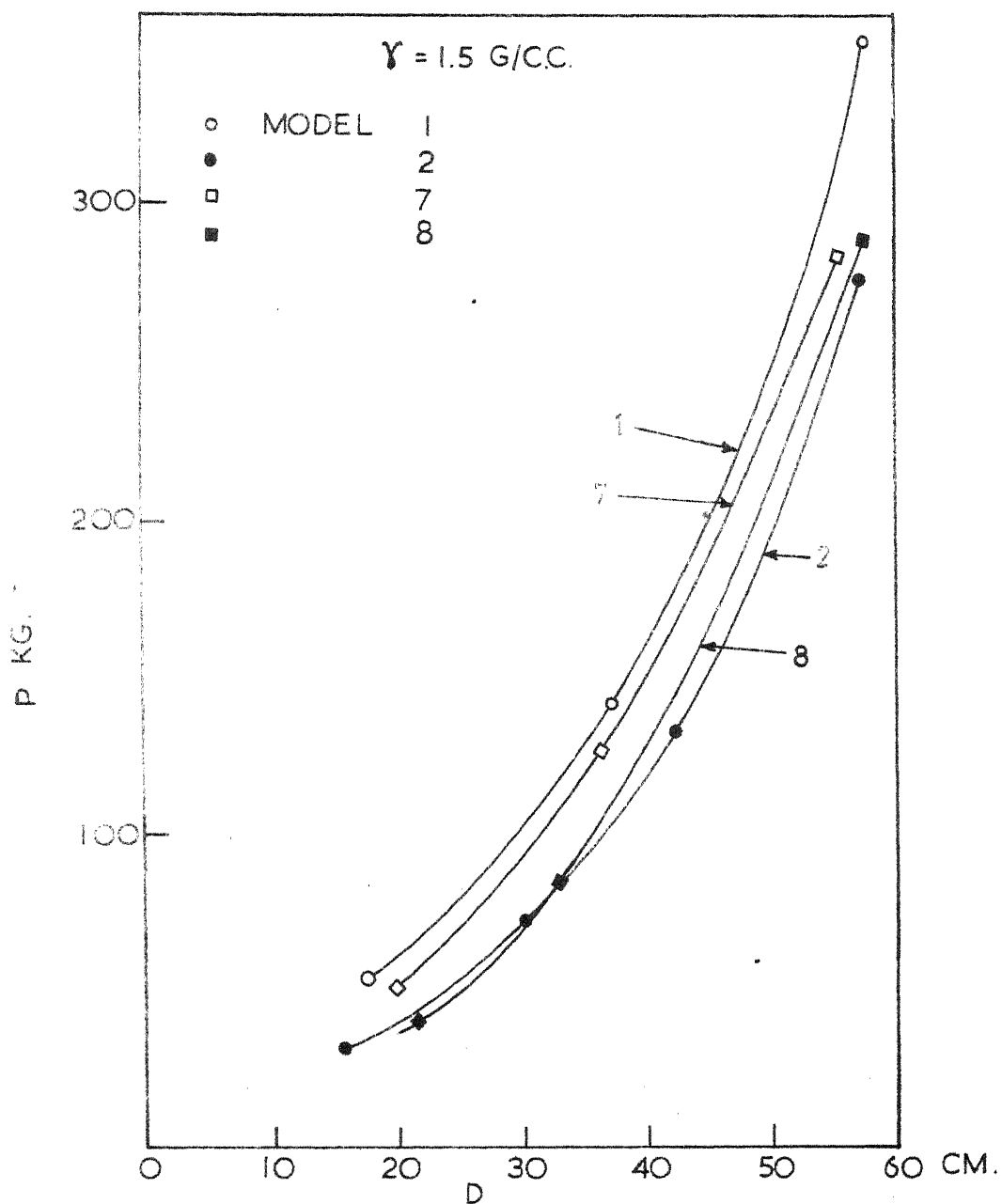


FIG. 4.1. PULL OUT FORCES IN MEDIUM SAND

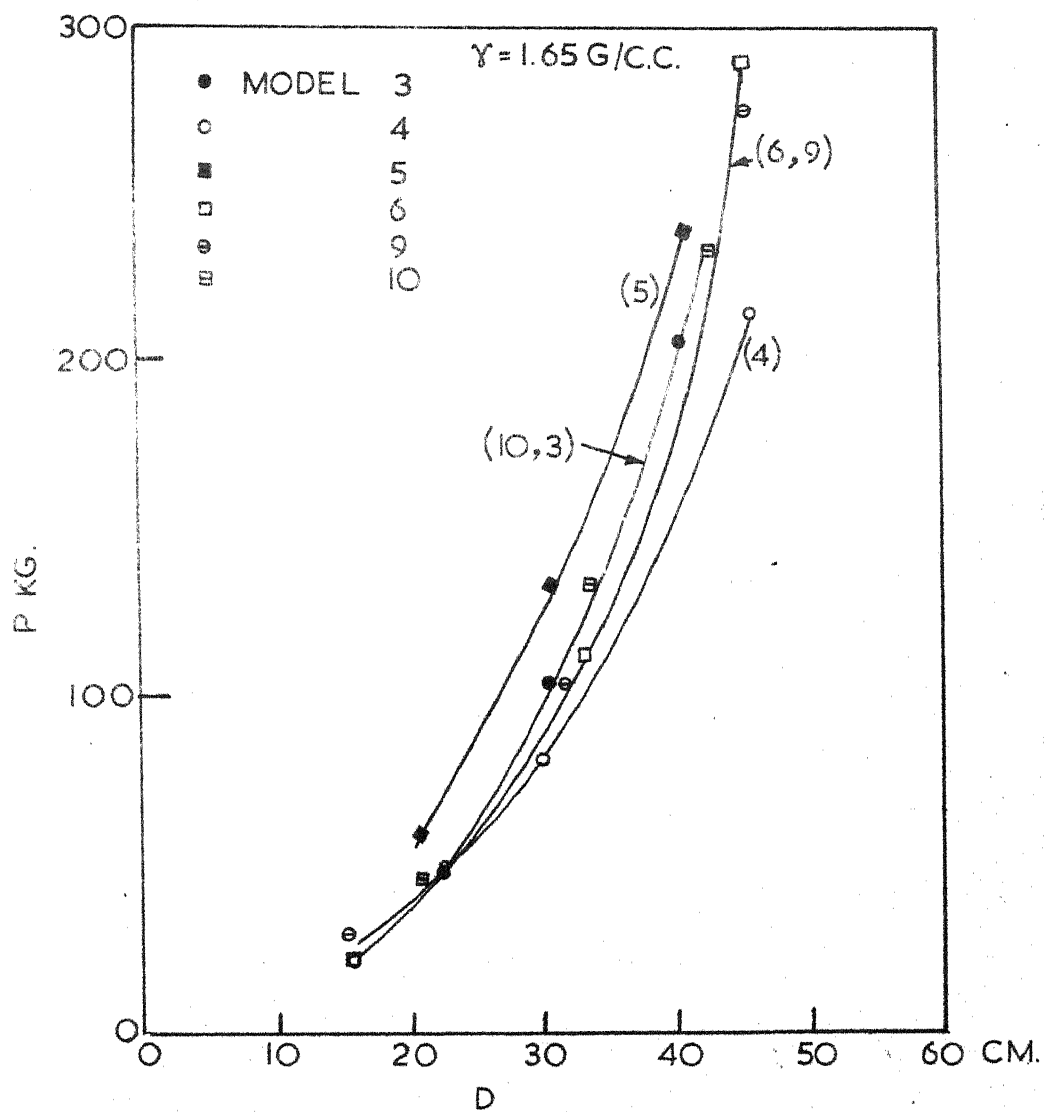
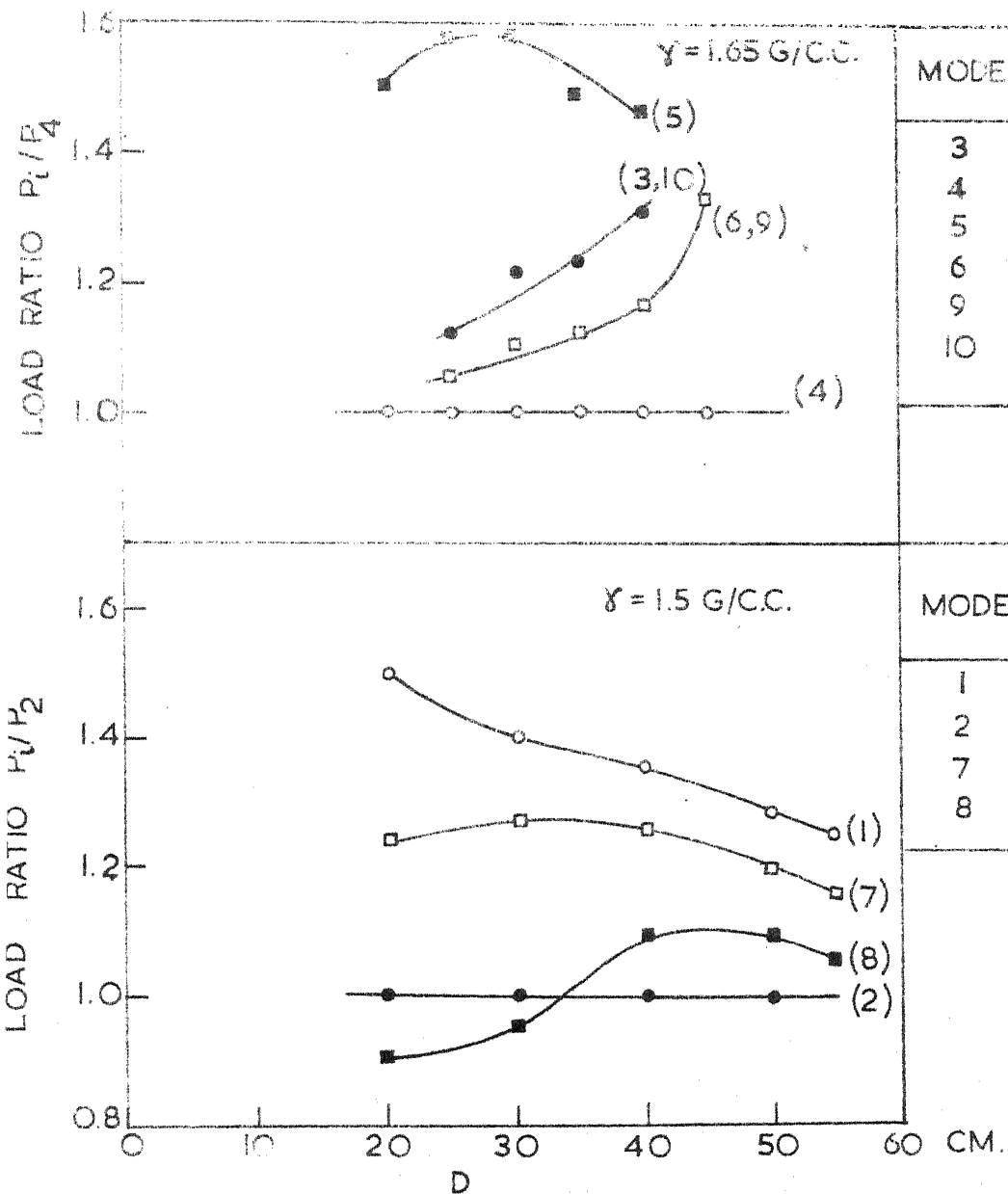


FIG. 4.2\_PULLOUT FORCES IN DENSE SAND



MODEL	$A_r = \frac{A_1}{A_4}$	$P_r = \frac{P_1}{P_4}$
3	1.78	1.33
4	1.00	1.00
5	2.26	1.69
6	1.27	1.27
9	1.27	2.02
10	1.5	2.13

MODEL	$A_r = \frac{A_1}{A_2}$	$P_r = \frac{P_1}{P_2}$
1	1.56	1.25
2	1.00	1.00
7	1.13	1.27
8	0.85	1.06

FIG 4.3 - COMPARATIVE LOAD RATIOS

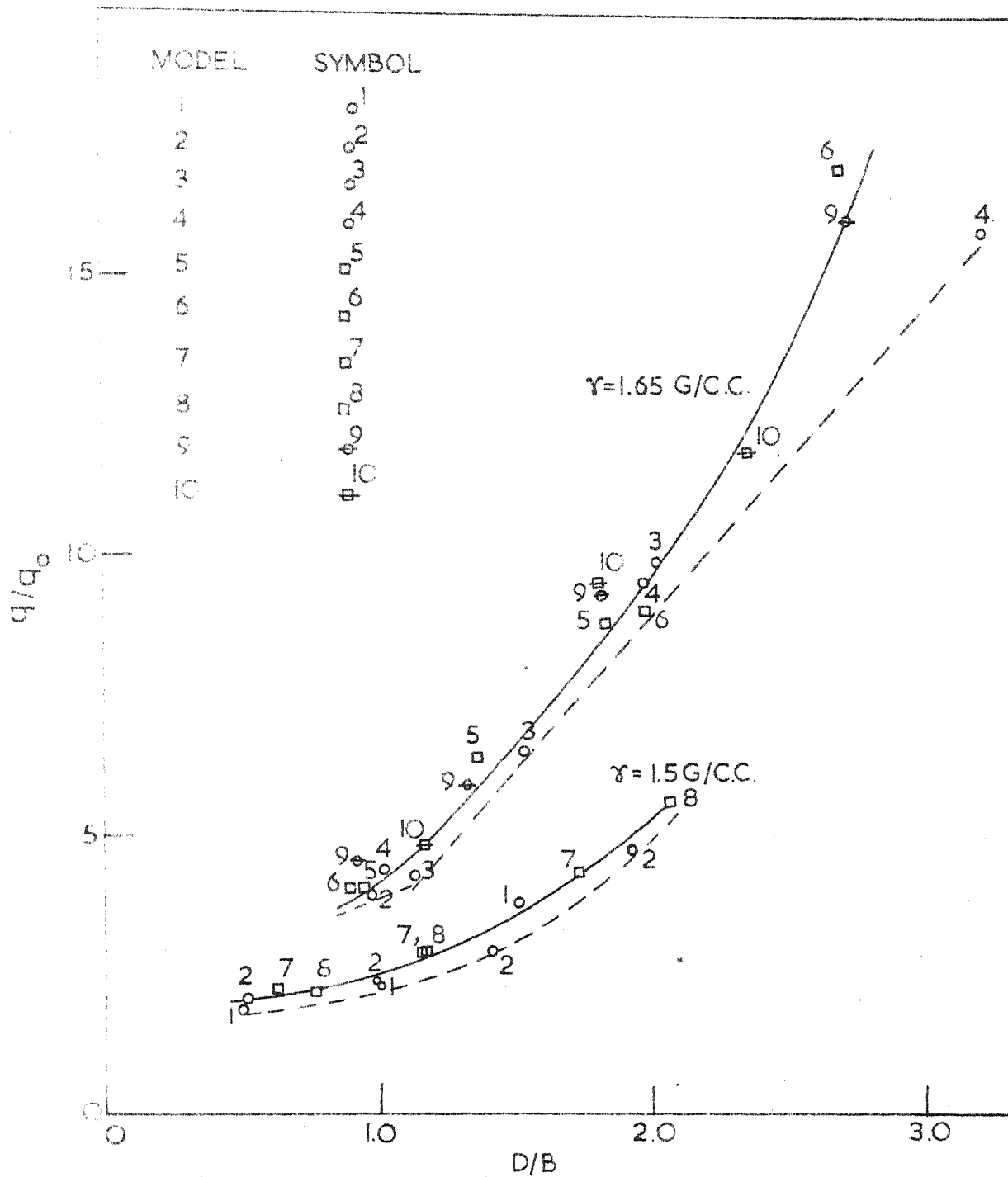


FIG.4.4\_A NON-DIMENSIONAL PLOT FOR PULL-OUT FORCES IN ALL TESTS

## CHAPTER 5

### CONCLUSIONS AND RECOMMENDATIONS

#### 5.1: CONCLUSIONS:

Large number of tests are necessary before firm conclusions can be generalised. However, for the fine variety of cohesionless soil, and in the ranges of  $D/B < 3$ , the following conclusions are tentatively drawn from the limited number of tests conducted:

1. For ordinary sands, Balla's values give lower than actual pull-out force values at shallow depths and very high values at greater depths. Matsuo's computations generally give lower values.
2. For dense sand, both Balla's and Matsuo's analyses grossly underestimate the pull-out strengths of the soil.
3. No apparent advantage is derived by special shapes as in models 9 and 10.
4. A rectangular footing with  $L/B = 1.5$  appears to be a good choice for ordinary sands, and a relatively smaller circular footing for dense sands.
5. Tests in each density of soil appear to follow a common trend (one curve for each density) of behaviour in the nondimensional plot  $\frac{D}{B}$  vs.  $\frac{q}{q_c}$ .

This is considered a significant result, because if further work on these and other densities show similar results, all the complicated analyses and formulae can be replaced by simple design equations in a non-dimensional form, depending only on the in-situ over-burden pressure of the soil; the shape factor or the effect of shape being indicated by small variations shown by the lower (and upper) bound lines.

6. The rupture surface is probably a log spiral from the footing top edge to the ground surface.
7. Compaction and densification of sand increases the pull-out force  $P$  considerably.
8. A minimum depth of embedment may perhaps be advisable to mobilise the elastic properties of soil to a significant extent.

## 5.2: RECOMMENDATIONS:

The subject of pull-out resistance of foundations is gaining in importance in recent times; nevertheless, analytical and experimental work on the topic is quite limited.

Further work on pull-out strengths of foundations may include studies on the following lines:-

1. Effect of water-content in soil,
2. Effect of the material of the footing and its roughness,

3. Effect of grain-size, and/or uniformity of sand,
4. Variation of strengths vis-a-vis the shear parameters of soil,
5. Effect of the mode of application of load, and,
6. Study of uplift resistance for different inclinations of load.

It is also necessary to develop suitable analytical theories, since the two analyses available are found to be grossly inadequate. Suitable theoretical analyses have also to include the pull-out force -

- i) in nonhomogeneous soils, including layered deposits,
- and ii) in anisotropic soils.

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